CREDIT RISK AND CREDIT RATIONING

By DONALD R. HODGMAN

I. Approaches to credit rationing, 258. — II. The influence of credit risk on loan payoff, 259. — III. Implications for lender behavior and borrower access to credit, 267. — IV. The central bank's influence, 275.

I. APPROACHES TO CREDIT RATIONING

Credit rationing is a much debated phenomenon. Although practical borrowers and lenders long have regarded credit rationing as a fact of experience, economists of a more analytical persuasion have been reluctant to accept it at face value because of their difficulty in providing a theoretical explanation for the phenomenon which is consistent with the tenets of rational economic behavior. Why should lenders allocate credit by non-price means and thus deny themselves the advantage of higher interest income? Explanations which meet this difficulty are those which attribute credit rationing to sticky interest rates associated with oligopoly in credit markets and to ceilings on interest charges imposed by usury laws. But most corporate borrowers are exempt from the protection of the usury statutes, and sticky rates of interest usually rise without too much delay as credit conditions tighten. These considerations have led to the apparently prevailing view that credit rationing as a general phenomenon, while it does occur, is primarily a temporary expedient which lenders resort to only until they have made up their minds to raise the interest rate to a new equilibrium level following an increase in borrower demand or restrictive action by the monetary authority. This view denies to credit rationing any significant influence on credit availability other than in the fairly short run.¹

Recently there have been attempts to free credit rationing from

¹I am deeply indebted to Professor Colin R. Hylth of the Mathematics Department, University of Illinois for developing my intuitive surmises into the explicit mathematical reasoning of Section I. Professor Marvin Frankel read the manuscript and provided helpful criticism. Professor Thomas A. Yancey gave generously of his time in several prolonged discussions of probability measurements. It is a pleasure to acknowledge the support of The Merrill Foundation for the Advancement of Financial Knowledge for the larger study from which this article derives.

¹ For an explicit statement of this view see Monetary Policy and Management of the Public Debt: Hearings before the Subcommittee on General Credit Control and Debt Management, Joint Committee on the Economic Report, 82d Congress, 2d Session, Statement of Paul Samuelson, esp. pp. 697–98.
its dependence upon interest ceilings, whether voluntary or imposed, by suggesting that credit rationing is an aspect of lender attitude toward risk. To date, so far as I am aware, these attempts have not been able to cope with one particular difficulty: the principle that when other factors such as credit rating or maturity are less favorable, borrowers may succeed in obtaining the credit they wish by paying a higher rate of interest to compensate lenders for the less favorable risk features of the loan or investment. Once this camel’s nose is under the tent, interest appears to regain much of the authority that the concept of credit rationing seeks to deny to it. If a determined borrower can always obtain funds by agreeing to a sufficiently high rate of interest, he can be denied credit only by quoting him a rate which he regards as prohibitive. This, however, is not credit rationing in the specific sense but traditional interest rationing.

In the analysis which follows I have sought to explore further the implications of risk for the terms upon which money is loaned. My purpose is to provide a more general explanation for credit rationing which does not rely upon oligopolistic market structure or legal maxima to the interest rate, which is consistent with rational behavior along the lines of economic self-interest, and which is permanent rather than temporary in its effect for so long as the general credit situation which occasions it lasts. The implications of the analysis extend to any credit transaction. However, the institutional influences which have conditioned my thinking are those of commercial banks, so there may be some special assumptions imbedded in the approach of which I am not aware. The reader may wish to remind himself of this occasionally.

II. THE INFLUENCE OF CREDIT RISK ON LOAN PAYOFF

The attractiveness of a loan or investment to a lender depends upon various factors: the interest rate, risk, possible benefits of a long term customer relationship and so on. In this section we focus our attention upon the interest rate and risk and their expression in a probability distribution of possible payoffs which is logically implied by the yield-risk features of any loan or investment. We shall see

2. For an otherwise suggestive treatment of credit rationing as an aspect of lenders’ attitude toward risk which fails exactly at this stage in the argument see Ira O. Scott, “The Availability Doctrine: Theoretical Underpinnings,” The Review of Economic Studies, XXV (Oct. 1957). The dependence of Scott’s solution on interest ceilings is stated in fn. 4, p. 46.

3. Note that all features of a loan including customer relationship can ultimately be reduced to the dimensions of a payoff distribution in an appropriate dynamic analysis. I have not attempted this in the present static analysis. However, this consideration signifies that the logic of rational lender behavior is
that the yield and risk aspects of an investment are systematically interrelated and can be varied relative to each other by the borrower only within specific limits. In particular, given the credit rating of the borrower, risk of loss is an increasing function of the size of the loan while the expected value of possible gains and losses (as a measure of yield under risky conditions) is an increasing function of the amount the borrower promises to repay (inclusive of interest). However, the influence of the borrower's credit rating sets an upper limit to the expected value of the payoff while no such limit applies to the expected loss. These conclusions follow logically from some fairly simple and intuitively reasonable stipulations concerning borrower capacity to pay.

The purpose of this section is to establish the logical basis for these conclusions. Their implications for lender behavior and for borrower access to credit are explored in Sections II and III. A reader who is prepared to accept these conclusions as intuitively convincing can save time and effort by skipping at this point to Section II. The balance of the present section is for skeptics.

Suppose that a lender has applied the techniques of credit analysis to the financial affairs of a potential borrower and has summed up his (the lender's) confidence in the borrower's capacity to pay various dollar amounts by assigning to each potential payment, \((y)\), a number between zero and one to represent the probability which the lender associates with the event that the borrower would pay exactly that amount if he promised to do so. This information on the probability of the borrower's capacity to pay various hypothetical sums can be summarized in the form of a mathematical function. Let \(\phi(y)\) be the probability that the borrower will be able to pay the amount \(y\). Some very general restrictions on the nature of \(\phi\) can be suggested. Since it is certain that the borrower can pay nothing if he makes this promise, \(\phi(0) = 1\). The consideration that the financial resources of a specific borrower are not unlimited implies \(\phi(y) = 0\) for \(y\) sufficiently large. Moreover, it seems reasonable to assume that \(\phi(y)\) is nonincreasing (the probability of repayment should not increase with the size of the amount promised) and continuous as well as differentiable for most \(y\). Further, for a range of small \(y\) values, \(\phi(y)\) may be close to 1. There are infinitely many functions having these properties. The general appearance of such functions is illustrated in Figure I.

More broadly encompassed in the present discussion than might appear to be the case in view of the omission of customer relations, maturities and other dynamic considerations.
For any $y$ on Figure I we can read off the probability that the lender attaches to the event that the borrower would pay exactly that $y$ should be promise to do so.\footnote{The function, $\phi$, is assumed to be independent of the size of the borrower's promise to pay and of the size of the loan granted. There must be many instances where this is reasonable, although there may be other situations for which one or both of these assumptions would have to be dropped.} Let $s$ be the amount that the borrower promises to pay. The lender's attention will be confined exclusively to $s$ only if $\phi(s) = 1$. For $\phi(s) < 1$ the actual payoff may be less than $s$. Assume that the borrower pays $s$ if he is able and otherwise pays the greatest amount which he can. Let $Y$ be the amount the borrower actually pays. This is a random variable. The cumulative distribution function of $Y$ is:

$P(Y \leq y) = 1 - \phi(y), y < s.$

$P(Y = s) = \phi(s).$

\footnote{The function, $\phi$, is assumed to be independent of the size of the borrower's promise to pay and of the size of the loan granted. There must be many instances where this is reasonable, although there may be other situations for which one or both of these assumptions would have to be dropped.}
The appearance of this distribution function is illustrated in Figure II. The probability $P(Y \leq y)$ is nondecreasing in $y$, and at $y = s$ jumps to 1 if it has not already reached 1 (since the borrower will never pay more than $s$).

If $\phi(y)$ is almost everywhere differentiable for $0 < y < s$, the

\[ P(Y \leq y) \]

probability distribution of $Y$ can be given by the density $-\phi'(y)$ on $0 < y < s$, together with the non-zero probability on $s$:

\[ P(a < Y < b) = \int_a^b -\phi'(y)dy, \quad 0 \leq a < b \leq s \]

\[ P(Y = s) = \phi(s). \]

The appearance of this density is shown in Figure III. Thus the probability that $Y$ lies between any two limits less than $s$ and equal
to or greater than zero is shown by the area under the curve, while
the probability that \( Y \) equals \( s \) is simply \( \phi(s) \).

To understand the reason for our next step we must pause to
consider what aspects of the payoff function associated with a borrow-
er's promise to pay a particular sum the lender may regard as relevant.
One way to summarize the payoff function is to calculate its expected

\[-\phi'(y)\]

FIGURE III

value. There is, however, considerable cogency to the view often
expressed in the literature concerned with the analysis of risky choice,
that the lender (investor, gambler) faced with such a payoff distribu-
tion may not be indifferent to the dispersion of the distribution
around its expected value. Frequently some such measure of dis-
persion as the standard deviation or variance of the payoff distribu-
tion has been utilized to illustrate the influence of dispersion on the
utility of risky choices. Either of these measures is acceptable.
However, to emphasize the asymmetrical influence of payoffs which
imply gains versus those which imply losses, I prefer to use a measure
of the expected value of probable losses to be defined below. Accordingly, we shall proceed on the assumption that the aspects of the payoff distribution which the lender regards as relevant are the expected value of the entire distribution and the expected value of the probable losses. The use of these measures is intended to be suggestive rather than definitive.

![Figure IV](image)

The expected value of the payoff distribution depends upon the \( \phi \) function (illustrated in Figure I) and \( s \), the promise to pay made by the borrower. The expected value of \( Y \) is

\[
EY = sP(Y = s) + \int_0^s - y\phi'(y)dy
\]

\[
= \int_0^s \phi(y)dy \text{ using integration by parts.}
\]
The derivative of the expected value with respect to $s$ is

$$
\frac{d}{ds} EY = \phi(s).
$$

Accordingly, for a $\phi$ function similar to that in Figure I, the expected value of the payoff increases with $s$ at a decreasing rate and approaches a limiting value. This functional relationship between $EY$ and $s$ is illustrated in Figure IV.

![Diagram](image)

**FIGURE V**

A loss occurs when the payoff is less than the amount of the loan given the borrower. If the amount loaned is $a$ and the amount paid is $Y$, the loss $Z$ is

$$
Z = 0 \text{ if } Y \geq a
$$

$$
Z = a - Y \text{ if } Y < a.
$$

The expected value of probable losses depends upon the borrower's estimated capacity to pay (the $\phi$ function), his promise to pay ($s$),
and the size of the loan \(a\) made by the lender. Thus,

\[
EZ = \int_0^a (a - y)[-\phi'(y)]dy = a - \int_0^a \phi(y)dy, \ a \leq s
\]

\[
EZ = a - \int_0^s \phi(y)dy, \ a > s.
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
a_2
\]

\[
a_3
\]

\[
(EZ/a) = 1
\]

\[
\frac{EY}{a}
\]

\[
\frac{EZ}{a}
\]

\[
45^\circ
\]

\[
(EZ/a) = 1
\]

\[
s
\]

\[
a_1
\]

\[
FIGURE VI

Normally the lender will not make loans when \(a > s\), but it is useful to have \(EZ\) defined for that possibility to facilitate certain diagrammatic treatment below. It may be useful to point out that an increase in \(s\) after \(s = a\) does not affect \(EZ\) although it does increase \(EY\) until \(\phi(y) = 0\). This is apparent from the fact that the payoff function depends only on the \(\phi\) function and \(s\). Thus, for any
specific \( a \), say \( a_0 \), \( EZ \) declines as \( s \) increases from zero to \( a \) and thereafter remains constant for further increases in \( s \) as in Figure V.

III. IMPLICATIONS FOR LENDER BEHAVIOR AND BORROWER ACCESS TO CREDIT

We turn now to the implications of our analysis for lender behavior and borrower access to credit. From the relationships discussed thus far it is possible to determine the various combinations of expected value of payoff (\( EY \)) and expected loss (\( EZ \)) which confront a lender who considers making various size loans (\( a \)) in response to various promises to pay (\( s \)) on the part of a specific borrower. The derivation of this information is described in Figure VI. The northeast quadrant is identical to Figure IV except that \( EY \) for each \( s \) has been replaced by \( \frac{EY}{a} \) for each \( s \) and \( a \). Since \( EY \) increases with \( s \) until \( \phi(y) = 0 \), \( \frac{EY}{a} \) increases similarly with \( s \) for each value of \( a \). The southeast quadrant of Figure VI is identical with Figure V except that \( EZ \) has been replaced by \( \frac{EZ}{a} \) and possibilities for an additional loan (\( a_e \)) have been included.\(^5\) Dividing \( EY \) and \( EZ \) by the relevant \( a \) simply serves to emphasize the relationship of expected value and expected loss to the dollar amount of the loan. These are essential relationships when we come to consider the ability of the borrower to meet competitive market terms. The northwest quadrant of Figure VI shows the achievable combinations of \( \frac{EY}{a} \) and \( \frac{EZ}{a} \) for different loans (\( a \)) and promises to pay (\( s \)) for a borrower whose credit rating, (\( \phi \) function), has been determined. The southwest quadrant is simply a means to transfer the \( \frac{EZ}{a} \) values from the lower half of the vertical axis to the left half of the horizontal axis.

To have this information on our borrower's capacity to demand in a form more convenient for purposes of relating it to the terms upon which the lender offers credit we shall now transfer it to Figure VII which is one of the key figures in our analysis. In Figure VII combinations of \( \frac{EY}{a} \) and \( \frac{EZ}{a} \) which a particular borrower can attain by vary-

5. The ratio \( \frac{EZ}{a} \) for any \( a \) is 1 when \( s \) is zero.
ing his promise to pay (s) are shown as the locus of points along the various loan curves labeled \( a_j \) (where \( j = 0 \ldots n \)). Variations in \( s \) are implied but not explicitly shown in Figure VII. However, the relevant value for \( s \) can be ascertained by inserting the \( \frac{EY}{a}, \frac{EZ}{a} \) co-or-

\[ \frac{EY}{a} \]

\[ \frac{EZ}{a} \]

\( (\frac{EY}{a}) = 1 \)

\( (\frac{EZ}{a}) = 1 \)

\( S' \)

\( S'_1 \)

\( S'_2 \)

\( O \)

\( D \)

\( a_0 \)

\( a_1 \)

\( a_2 \)

\( a_3 \)

\( s = a \)

\( S' \)

\( S'_0 \)

\( O' \)

\( D' \)

\( M' \)

\( \frac{EZ}{a} = 1 \)

\[ (\frac{EZ}{a}) = 1 \]

\[ (\frac{EY}{a}) = 1 \]

**FIGURE VII**

dinates of any point on Figure VII into Figure VI and reading off the resulting required values for \( s \). The curve labeled \( (s = a) \) which slopes downward to the right is the locus of minimum expected losses at associated minimum values of \( EY \) for loans of different amounts. If there is any risk \( (\phi(y) < 1) \), the expected value at \( (s = a) \) is
always less than \((s = a)\), that is \(\frac{EY}{a} < 1\). Moreover, \(\frac{EY}{a}\) for \((s = a)\) decreases as \((s = a)\) and associated \(EZ\) increase. Therefore, the locus of minimum \(\frac{EZ}{a}\) and associated \(\frac{EY}{a}\) values falls further below the line \(\frac{EY}{a} = 1\) as \((s = a)\) increases. The maximum \(\frac{EY}{a}\) attainable by the prospective borrower for any \(a\) occurs when \(\frac{dEY}{da}\) becomes zero.

This maximum ratio for \(\frac{EY}{a}\) (shown by \(MM'\) in Figure VII) also declines as \(a\) increases.

Our next step is to introduce the borrower's demand schedule which expresses his willingness as distinguished from his capacity to meet the terms which the lender may require. The borrower's demand schedule appropriate to Figure VII is easily derived from a conven-
tional demand schedule for loanable funds. Suppose our borrower's conventional demand schedule is that shown by DD' in Figure VIII. (The reader is asked to ignore the supply schedules in Figure VIII for the present.) This schedule relates the size of the loan \((a)\) which the borrower will want to the contract rate of interest \((r)\) charged.

But \(r = \frac{s - a}{a}\). Therefore, any pair of \((r, a)\) from the demand schedule of Figure VIII implies a specific value for \(s\), the borrower's promise to pay. Given specific values for \(s\) and \(a\), specific values for \(\frac{EY}{a}\) and \(\frac{EZ}{a}\) are likewise determined. Thus, the borrower's conventional demand schedule is readily translated into terms of \(\frac{EY}{a}\) and \(\frac{EZ}{a}\) and can be represented on Figure VII. In making the translation one restriction must be observed. No matter how high the value for \(r = \frac{s - a}{a}\) to which the borrower will agree, the upper limit to the corresponding value for \(\frac{EY}{a}\) is determined by the borrower's credit rating (the \(\phi\) function). This limit is represented in Figure VII by the curve \(MM'\). Therefore, the borrower's effective demand cannot rise above \(MM'\). The demand schedule of Figure VIII translated into terms of Figure VII is shown by the dotted curve \(DD'\). Although the borrower might well wish to take larger loans at combinations of \(\frac{EY}{a}\) and \(\frac{EZ}{a}\) lying below and to the left of the lower end of the demand schedule in Figure VII, the lender will never lend at such terms (unless he is an avid gambler with little regard for odds), since they are always inferior to holding cash for which \(\frac{EY}{a} = 1\) and \(\frac{EZ}{a} = 0.7\).

The terms upon which the lender is willing to lend are shown by the supply curves (labeled \(SS_j, j = 0 \ldots n\)) in Figure VII. Only one such \(SS_j\) curve is in effect at any one time. We shall now examine the factors which determine the shape and position of these \(SS_j\) curves as well as which curve is operational at a particular time. These factors include the subjective preferences of the lender for yield versus

6. We shall not explore the analytical determinants of borrower demand under risk and uncertainty, although this would improve the symmetry of the discussion.

7. This statement ignores price changes in the markets for goods and services.
risk of loss, the terms upon which he can exchange risk of loss for yield as established by market conditions (i.e., borrower demand and competing supply), his total supply of investable funds, and his liability position which must be related to his asset position before the implications of his asset position for yield and risk can be determined. The influence of a lender's liability position has been neglected in the formal literature of investment choice. This is not the occasion to attempt to repair this neglect. Perhaps it will be intuitively obvious to the reader that two lenders with different liability structures may hold identical collections of assets and yet face quite different yield-risk situations. The simplest way to handle this problem in the present context is to incorporate the effect of the lender's liability structure in his indifference curves. This means, of course, that two lenders with identical subjective attitudes toward risk and yield are likely to have quite different preference patterns for assets. (Differences in liabilities are probably a far more important cause of differences in institutional asset preferences than the subjective attitudes of institutional managers.) Once we have incorporated the influence of liability position in the indifference curves, we are entitled to concentrate on the available asset characteristics and combinations of assets.

The indifference curves (labeled $I_1$, $I_2$, etc.) in Figure IX represent our lender's preference pattern for yield versus risk in his total asset collection as conditioned by his subjective preferences and his liability position. Note that Figure IX is intended to represent the risk-yield implications of the lender's total asset position in contrast to the marginal changes in that position caused by a single borrower as reflected in Figure VII. (Note also, that the alternative risk-yield combinations of Figure IX do not represent the ultimate risk-yield position of the lender. For this, the influence of the liability position must be made explicit.) The influence of market conditions and the volume of total investable resources available to the lender are represented by what we may call "opportunity curves" labeled $O_1$, $O_2$, etc. Only one of the opportunity curves is in effect at any one time. For given market conditions an opportunity curve can be derived by considering the terms upon which the lender can extend credit to the various borrowers who seek accommodation.\(^8\) On the

---

8. There is a problem of interdependence among payoff functions which I propose to ignore. It can be dealt with formally by replacing EZ by the variance of the payoff distribution (for mathematical convenience) for individual borrowers, introducing covariance measures between different distributions, and following methods outlined by Harry Markowitz in "Portfolio Selection," The Journal of Finance, March 1952.
conventional assumption that borrowers' demand curves are downward sloping and their credit ratings subject to deterioration as the amounts they borrow increase, the slope of the opportunity curves must diminish as they extend further to the right. This simply reflects the fact that the lender must accept less favorable terms to expand his volume of loans. Moreover, in a competitive market no borrower will pay better than the worst terms.

**FIGURE IX**

Given the opportunity line (borrower demand) the positions available to the lender depend upon the volume of investable resources in his possession. This magnitude is not shown explicitly in Figure IX, but its effect is indicated by the solid portion of the opportunity lines. The lender has sufficient investable resources to go out a given opportunity line to the point where the line becomes dashed (- - - -). At that point the lender is fully invested (that is, cash is zero) and is
powerless to extend additional credit. The optimum risk-yield combination for the lender is shown diagrammatically as the highest indifference curve attainable (by intersection or tangency) by traveling along the solid portion of the existing opportunity line. The optimum point in Figure IX will normally be a tangency rather than an intersection. This follows from the consideration that a lender will usually retain some cash for his own transactions and precautionary requirements. Therefore, his fully invested position (cash = zero) will have to be further out on the relevant opportunity line than his optimum position. But this implies a tangency optimum. However, an intersection optimum is possible if the lender is restrained from reducing his (as he regards it) surplus cash because of a minimum cash requirement imposed by law or regulatory authority.

In a competitive market our borrower must pay the going rate for funds. Let $A$ (where $A = \sum a_j$ and $j = 1 \ldots n$) represent total earning assets of the lender. Then \( \frac{\Sigma EV}{\Sigma EZ} \) at (or in the neighborhood of) the optimum point in Figure IX are the \( \frac{EV}{EZ} \) terms which our borrower must meet to qualify for a loan.

The loans and terms available to the particular borrower to which Figure VII applies are shown by the relevant $SS'_y$ curve on that diagram. Let us suppose that the tangency optimum in Figure IX establishes $SS'_y$ as the operational supply curve in Figure VII.\(^9\) Then our borrower is able to qualify for any loan between zero and $a_1$. Given supply curve $SS'_y$ our borrower cannot qualify to borrow more than $a_1$ because his credit rating ($\phi$ function) does not warrant a larger loan at market terms. Assuming $s \geq a$, the loan requested carries with it an irreducible risk of loss indicated by the location with respect to the $\frac{EZ}{a}$ axis of the vertical portion of the respective $a$ curve.

The maximum $\frac{EV}{a}$ or $EV$ which the borrower can generate to offset that irreducible risk also is a function of the $\phi$ function (and $s$). Thus, the area of effective borrower demand is bounded by the $MM'$ curve,

\(^9\) If the optimum in Figure IX occurs at an intersection rather than a tangency (due to an imposed minimum cash requirement) the relevant $SS'_y$ curve in Figure VII will intersect the vertical axis at a point where $\frac{EV}{a} > 1$ since the marginal utility of available additional loans will exceed the marginal utility of cash to the lender.
the relevant $SS'$ curve, and the vertical axis. Clearly, as changes in
the optimum point on Figure IX cause the operational $SS'$ curve of
Figure VII to rotate (or shift) northwest, our borrower's ability to
qualify for loans is steadily and inescapably decreased. In particular,
the maximum loan for which a particular borrower can qualify declines
as the minimum qualifications in terms of $EY/a$ and $EZ/a$ increase.
If the borrower's willingness to demand falls short of his capacity, the
loan agreed upon is shown by the intersection of $SS'$ and $DD'$ in
Figure VII.

It is instructive to look at the analogue of the supply and demand
curves of Figure VII in conventional supply and demand terms.
Since we already have the demand equivalent in Figure VIII we need
only translate the supply curves of Figure VII into their equivalent
expression in Figure VIII. This is readily done by translating the
$\frac{EY}{a}$ and $a$ values of points located on a supply curve of Figure VII
into their interest rate and $a$ equivalent for the particular borrower.

Given $a$ and $\frac{EY}{a}$, the $s$ required of the borrower can be read from the
northeast quadrant of Figure VI thus determining $r = \frac{s - a}{a}$. The
conventional supply curves analogous to the two-dimensional supply
curves $SS_0'$, $SS_1'$ and $SS_2'$ in Figure VII are identically labeled in
Figure VIII. It must be emphasized that these conventional supply
curves of Figure VIII are specific to one particular borrower (or, at
least, one particular $\phi$ function). We may note that supply curve $SS_0'$
in Figure VIII is interest-elastic both above and below its inter-
section with $DD'$; the borrower could obtain additional funds if he
were willing to increase his promise to pay, ($s$). At $a_2$, however, $SS_0'$
becomes totally interest inelastic. The borrower's capacity to borrow
has reached its limit. Should $SS_1'$ replace $SS_0'$, our borrower would
find himself borrowing at absolute capacity at $a_4$. He would be
powerless to borrow more, since his credit rating would prevent him
from increasing $EY$ to compensate the lender for the unavoidable
increase in $EZ$ which would accompany an increase in $a$. A further
shift to $SS_2'$ in Figure VIII would further reduce the maximum loan
available to the borrower from $a_4$ to $a_6$. The usual interpretation of
the intersection of $DD'$ and $SS_2'$ leads to the erroneous conclusion
that the contract rate of interest for $a_3$ is $r_1$. In fact, however, the
contract rate for $a_6$ is $r_2$. There are two reasons for this. The borrower
will receive $a_6$ either by promising $r_1$ or $r_2$ and, hence, will promise the
less, $r_2$. The lender, thinking in terms of $\frac{EY}{a}$ and $\frac{EZ}{a}$ for $a_0$, will regard these values as identical for rates $r_1$ and $r_2$ and will be indifferent between the two contract rates of interest.

Whenever the borrower's demand curve intersects with a vertical portion of the relevant supply curve, the particular borrower will be unable to obtain additional borrowed funds by promising to pay additional interest. Moreover, as the supply curve shifts to the left and upward, the borrower will encounter progressively more stringent restrictions on the supply of funds which he will be unable to overcome by offering to pay more interest. However, another borrower with a superior credit rating ($\phi$ function) may continue to be able to borrow as much as he wishes and even may not be required to pay much additional interest to meet the higher qualifications imposed by the lender. The borrower with the inferior credit rating will regard "availability" as more important than interest in determining his access to borrowed funds.

It is this combination of circumstances which has come to be known as "credit rationing." Clearly, however, the "rationing" involved here is quite a different phenomenon from that associated with a regulated ceiling price. Two points of difference may be emphasized. First, the lender is not denying himself (or being denied) an advantage (higher interest) which he normally seeks, but is behaving rationally in the face of risk. Second, the supply of loanable funds to some borrowers remains interest-elastic even after becoming completely inelastic to others. In this connection we may note that the lender's indifference curves in Figure IX as well as the supply curves in Figure VII retain their elasticity even after credit is rationed. Indeed, since there is always the "unsatisfied fringe of borrowers," to make credit rationing depend upon interest inelastic supply without distinguishing among different borrowers would make our theory contradict the empirical evidence that there are always some borrowers who can obtain additional accommodation at higher rates of interest.

IV. THE CENTRAL BANK'S INFLUENCE

There are numerous topics in monetary theory which can usefully be explored from the vantage point we have now reached. In the interests of brevity I shall confine my remarks to establishing that central bank policy can alter the location of the optimum in Figure IX and, hence, the terms available to the individual borrower in Figure VII or VIII. Then I shall comment on the implications of this
process for the ability of the central bank to restrict credit without "excessive" increases in the effective rate of interest on government bonds.

Suppose the lender, a commercial bank, happens to be at an optimum point \( P_1 \) in Figure IX. Now imagine that the central bank raises reserve requirements. This has no initial effect on the market value of the bank's total assets (its wealth position), but it does require the bank to sell some earning assets to increase its cash (including reserve balances). Three effects follow: (1) the volume of earning assets which the bank can hold is reduced; it must move back along the opportunity curve toward the origin; (2) to the extent that the sale of bank assets to replenish its cash position reduces asset prices, interest rates are raised and a new opportunity curve (say \( O_2 \)) is created with slope higher than the original opportunity curve; (3) to the extent that interest rates rise and asset prices fall the market value of the bank's total resources falls; since the value of its total liabilities (principally deposits) remains virtually unchanged, it is now in a more risky position than before. In Figure IX such an increase in risk has to be shown by an increase in the slopes of the indifference curves such as represented by \( I_0 \). All these effects reinforce each other to produce a new optimum point in Figure IX (say at \( P_2 \)) for which the ratio \( \frac{E_Y}{E_Z} \) per dollar loaned or invested is higher than before. This is equivalent to moving to a higher supply curve in Figures VII and VIII and thus to requiring higher terms from the borrower.

A similar chain of consequences follows central bank sale of government securities in the open market. If our bank is typical, this action will both reduce its reserve balances and its deposit liabilities. The net effect will be to reduce its cash position and hence its liquidity by a greater proportion than its liabilities and hence to increase risk. This will steepen the slope of the indifference curves in Figure IX at the same time that it reduces total investable resources. Moreover, the sale of government securities by the central bank will have raised interest rates and depressed asset values. This raises the slope of the opportunity line and reinforces the decline in investable (indeed, total) resources in the hands of the bank. Once again the ratio \( \frac{E_Y}{E_Z} \) per dollar loaned or invested will be higher at the new optimum than at the old and terms to borrowers will have risen.
What insight does this analysis permit into the ability of the central bank to restrict credit expansion without, or with only a modest, rise in interest rates on government securities? Clearly, any restrictive action by the central bank, ceteris paribus, must produce some rise in interest rates on government securities, although the rise may be almost imperceptible if conditions are favorable. The extent of the rise in interest which must accompany a given degree of credit restriction will depend upon the character and composition of aggregate borrower demand. The larger the proportion of marginal borrowers in the aggregate demand (those who do not wish or are unable to raise \( \frac{EV}{EZ} \) when terms increase) the more effectively will credit be restricted for a given rise in interest on government securities. The larger the proportion of determined prime borrowers the greater the reliance that will have to be placed on interest rationing as opposed to credit rationing to restrict credit. Of course, the higher interest rates go, the more numerous are the borrowers who become marginal in terms of credit rationing (risk considerations) regardless of their willingness to commit themselves to higher interest charges. Ultimately every borrower is limited by his capacity rather than his willingness. Accordingly the higher interest rates go, the more pervasive will become the influence of credit rationing. More and more borrowers will find that “availability” rather than interest determines their access to credit. Indirectly this will affect even the prime borrowers, since the markets they sell in are dependent on effective demand and this may come to be strongly influenced by the restrictions that credit rationing places on their customers.\(^1\)

The rise in interest rates that accompanies credit restriction (or increase in borrowers' demand with money supply constant) can, of course, be restrained by the presence of interest ceilings, either those which are due to the oligopolistic structure of credit markets or those due to usury laws. The presence of such ceilings will accentuate the non-price rationing of credit. However, the credit rationing occasioned by such ceilings lasts only so long as the ceilings are maintained. (Usury ceilings may be presumed to last longer than oligopolistic ceilings, but most corporate borrowers are exempt from usury

---

1. Note that if the central bank tightens credit with member bank reserves constant, for example by buying short term government securities and selling long terms, it may exercise its influence directly on the velocity rather than the volume of the money supply, since it may impede the transfer of idle to active balances while leaving money supply unchanged.
ceilings.) The credit rationing which stems from borrowers' credit ratings, however, is permanent for so long as the general credit situation which gives rise to it lasts. It is this more general phenomenon of credit rationing, long neglected in monetary theory, which has been the central issue in this article.

UNIVERSITY OF ILLINOIS