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Credit Bureau Policy and Sustainable Reputation Effects in Credit Markets

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Welfare-increasing reputation effects arise in credit markets when adverse selection gives rise to borrower reputation formation incentives that mitigate moral hazard problems. This paper shows that welfare stemming from reputation effects will diminish over time as the private information of borrowers is revealed to lenders in the form of lengthening credit histories. Aggregate borrower welfare may therefore decrease over time unless reputation effects can be sustained. Restricting a lender’s access to a borrower’s credit history via credit bureau policy is shown to be one method of sustaining reputation effects and preventing a decline in welfare.

Introduction

A relatively large literature has emerged that examines how moral hazard and adverse selection reduce aggregate participant welfare in credit markets. Moral hazard arises because lenders of capital cannot directly monitor the actions of borrowers and thus are not able to condition the financing agreement on such actions. Because of the moral hazard, borrowers have an incentive to choose excessively risky projects (e.g. Myers 1977, Stiglitz and Weiss 1981) or to supply too little effort to their investment (e.g. Brander and Spencer 1989, Innes 1990, 1993). Adverse selection, on the other hand, occurs when borrowers have private information about the distribution of returns from their investment. In this asymmetric information case, high-quality borrowers are forced to cross-subsidize low-quality borrowers because lenders cannot distinguish among the various types when a contract is being negotiated. Adverse selection has pronounced distributional effects and may also result in an inefficient level of aggregate investment (e.g. Stiglitz and Weiss 1981; DeMeza and Webb 1987, 1988; Innes 1991).

Comparatively few studies have modelled the joint existence of moral hazard and adverse selection or the repeated interaction of borrowers and sellers. Both of these features are common to most credit markets and when taken together give rise to an important phenomena referred to as reputation effects. Simply stated, reputation effects arise because, through choosing a safer investment or supplying more effort in a given period, a borrower can lower the probability of default. Making payment rather than defaulting provides future benefits for the borrower by altering lenders’ beliefs. Because of the uncertainty about a borrower’s type, lenders will upgrade their beliefs about the quality of borrower they are dealing with when payment was made the previous period and will downgrade their beliefs when default occurred. High-quality borrowers therefore expect to receive a lower cost of capital than low-quality borrowers and are not as likely to have their credit rationed or to be
excluded from the credit market. Both moral hazard and adverse selection are required for reputation effects to emerge, because without the former borrowers are not in a position to affect the distribution of their returns, and without the latter credit histories play no role in the setting of interest rates.

Because of the importance of moral hazard and adverse selection in most credit markets, as evidenced by emphasis that is typically placed on credit histories, reputation effects are likely to be quite pronounced in the real world. For many inexperienced entrepreneurs, building up a good credit history through hard work and conservative investing is an important objective. This is not surprising, given that borrowers with unfavourable credit histories (e.g. past bankruptcies, delinquent payments and forced collections) typically have much poorer access to credit and at poorer terms. Bankruptcy in particular has harsh consequences since it normally involves a temporary ban from borrowing and a relatively higher cost of capital after borrowing resumes. And in today’s information age, it is rare for borrowers to hide from their credit background because credit histories are usually well documented by credit bureaus and are readily accessible by potential lenders at low cost.

This paper shows that reputation effects in credit markets are welfare-improving because they serve to offset some of the loss stemming from moral hazard by moving the equilibrium closer to first-best. Specifically, borrowers will ignore at least some of the short-run incentives to choose an excessively risky project or reduce effort to a sub-optimal level because of their concern over their credit history. Borrowers are rewarded for choosing a safer project or supplying more effort through a lower and more efficient cost of capital and as a result achieve higher welfare.

The main contribution of this paper, however, is to show that, although reputation effects are desirable from an efficiency perspective, they are generally not sustainable over time for a given cohort of borrowers. The reason is that, as credit histories lengthen, lenders become increasingly informed about the types of borrowers with whom they are dealing. Reputation effects are strongest when lenders are the most uncertain about a borrower’s type since it is at this point that lenders are willing to adjust their beliefs the most when new information is received. Diminishing reputation effects in turn imply a possible decline in aggregate welfare over time for a given cohort of borrowers. Policies that restrict the flow of information from borrowers to lenders may therefore be desirable from a social efficiency perspective because such policies can sustain reputation effects. Policies of this sort are quite common in most industrialized countries. For example, in Canada credit bureaus are restricted from releasing bankruptcy information about a borrower that is more than six years old.

In related work, Holmstrom (1982) and Macleod and Malcolmson (1988) have examined reputation effects in labour markets. Holmstrom uses reasons similar to those presented above to argue that reputation-related incentives steadily decline as information about a manager’s type is revealed over time. In contrast to Holmstrom’s analysis and the hypotheses of this paper, Diamond (1989) argues that reputation effects in credit markets strengthen rather than weaken over time. The inconsistency between Diamond’s results and the current results are due to structural differences in the respective models that can be explained as follows. In Diamond’s model, high-risk borrowers are gradually excluded from the credit market over time as their types become revealed to
lenders. The increasing concentration of low-risk borrowers results in a decline in the average cost of capital, and this decline is sufficient to induce a certain type of borrower to gradually switch from choosing a high-risk to a low-risk project. In the current analysis borrowers are never excluded from the credit market, so an analogous type of interest rate effect is not present.

Moral hazard is modelled as borrowers supplying effort to one particular project rather than choosing from among a number of alternative risky projects. Adverse selection is modelled as borrowers having different disutility of effort functions with a key parameter that is private information. This particular specification was chosen primarily because it allows for easier analysis than alternative specifications. A number of realistic features of credit markets are also assumed away in order to keep the analysis focused exclusively on reputation effects. For example, there is no borrower learning or capital accumulation over time. This assumption implies that, for a given level of effort and cost of capital, experienced borrowers are just as likely to default as inexperienced ones.

In the next section the equilibrium of a T-period lending game is characterized. Section II identifies the reputation effects, and Section III shows why these effects diminish over time. Section IV discusses credit bureau policy and the trade-offs involved with restricting information in order to sustain reputation effects. A simple example is constructed to illustrate these trade-offs. Section V concludes.

I. T-PERIOD REPUTATION GAME

Assumptions
The base model developed in this section builds on the single-period model of Brander and Spencer (1989) that examines the theory of the firm. The industry consists of a large number of borrowers and lenders of capital who behave competitively and who are risk-neutral. At the beginning of each of the T periods, each borrower obtains exactly one unit of capital from an arbitrarily chosen lender and uses the borrowed funds to purchase productive factors. Borrowers combine the purchased factors with their own labour and produce a good that is sold at the end of the period. Borrowers use the sales revenue to repay lenders the amount specified in the financing agreement.3

Borrowers are identical except with respect to a disutility of effort parameter, \( \theta \). Let \( \theta_i \) characterize the \( i \)th type of borrower; \( \theta_i \epsilon \{ \theta_1, \ldots, \theta_n \} \), where \( \theta_i > \theta_j \) if and only if \( i > j \). The disutility received by the \( r \)th type (measured in money-metric terms) when exerting effort \( e, e \epsilon [0, \bar{e}] \), equals \( c(e; \theta_r) \). This function is assumed continuous, twice differentiable and convex in effort (i.e. \( c_e > 0 \) and \( c_{ee} > 0 \)), and has the property that \( c(0; \theta) = 0 \).4 In addition, \( c_e(e; \theta_r) < c_e(e; \theta_j) \) if and only if \( \theta_r > \theta_j \); that is, a higher value of \( \theta \) corresponds to a borrower with a comparatively lower disutility of effort and hence a harder worker in the equilibrium. Adverse selection exists because the value of \( \theta \) is private information for each borrower. Moral hazard exists because lenders cannot observe the borrower's choice of effort after the loan contract has been negotiated.
Let the borrower's revenues at the end of each period be described by a continuous and twice differentiable function \( \pi(e, \omega) \) defined over \( e \) and a random disturbance variable, \( \omega \). For each borrower and for each time period, \( \omega \) is assumed to be an independent draw from a stable distribution function \( F(\omega) \) defined on the unit interval \([0, 1]\). The corresponding density function is \( f(\omega) \). Assume that \( \pi(e, \omega) \geq 0 \) and \( \pi(0, \omega) = \pi(e, 0) = 0 \) for all \( e \in [0, \bar{e}] \) and \( \omega \in [0, 1] \). Hence there exists a positive probability that a borrower's revenues will fall short of the contracted loan repayment, regardless of the level of effort he exerts. Also assume that \( \pi_e > 0, \pi_\omega > 0, \pi_{ee} < 0, \) and \( \pi_{e\omega} > 0 \) for all \( e \in (0, \bar{e}) \) and \( \omega \in (0, 1) \). The first two restrictions imply that more effort and larger values of \( \omega \) result in higher revenues for given values of the corresponding variable. The last two restrictions imply that the marginal productivity of effort is lower the larger is \( e \) but is higher the larger is the realization of \( \omega \).

The loan contracts written by lenders are of the standard debt form; that is, the lender's return equals \( \min\{\pi(e, \omega), 1 + r\} \) where \( r \) is the contracted rate of interest.\(^5 \) Loan contracts are costlessly enforced by an outside party, implying that the payment is always made when the borrower's revenues equal or exceed the scheduled payment. Lenders lend out funds provided that their expected return at interest rate \( r \) equals or exceeds their opportunity cost of funds (assumed to be constant), \( \rho \). For the purpose of a credit history, default is said to occur when the lender receives less than \( 1 + r \).

Define \( \tilde{\delta}(r) \) as the level of effort that just ensures that the borrower defaults with probability 1; that is, \( \pi(\tilde{\delta}(r), 1) \equiv 1 + r \). Hence there exists, for interest rate \( r \) and effort level \( e \in [\delta^0(r), \bar{e}] \), a unique state of nature \( 0 < \omega^* \leq 1 \), which ensures that a borrower's revenues are just sufficient to cover his debt obligations. The variable \( \omega^* = \omega^*(e, r) \) is implicitly defined by the identity \( \pi(e, \omega^*(e, r)) \equiv 1 + r \) when \( e \in [\delta^0(r), \bar{e}] \) and \( \omega^*(e, r) = 1 \) when \( e \in [0, \delta^0(r)] \). Total differentiation yields \( \omega^*_e = -\pi_e/\pi_\omega < 0 \) and \( \omega^*_r = 1/\pi_\omega > 0 \) for \( e \in [\delta^0(r), \bar{e}] \). These two results imply that (a) for a given interest rate, the more effort a borrower exerts the lower the probability that his revenues will be less than \( 1 + r \) (i.e. default occurs) and (b) for a given level of borrower effort, the higher the contracted rate of interest, the higher the probability that default occurs.

The above assumptions imply that the lending relationship between borrowers and lenders can be modelled as a repeated game. To simplify the strategy space in this repeated game, borrowers and lenders are limited to use single-period, rather than multi-period, debt contracts.\(^7 \) Hence borrowers are free to switch lenders, and vice versa, after each period. Diamond justifies single-period contracting by assuming that new lenders displace old lenders at the beginning of each period. Another important assumption is that it is too costly for lenders to find out anything about borrowers other than whether or not the borrower defaulted in each of the preceding periods. This assumption effectively precludes borrowers from signalling their type by accepting an interest rate that is higher than the minimum required rate because future interest rates cannot be conditioned on such signals.

The incomplete information nature of the relationship implies that the relationship should be analysed as a reputation game analogous to the one described by Kreps and Wilson (1982) and Milgrom and Roberts (1982). The equilibrium concept is perfect Bayesian equilibrium (PBE). A PBE can be described as a profile of strategies and beliefs such that, for each period of the
game and for each possible information structure, each agent plays optimally from that point onward, given his or her belief about the other players' type and given the strategy of all other players. In addition, beliefs must be updated rationally (i.e. using Bayes' rule whenever possible).

The more specific structure of the game is as follows. Many identical lenders compete by offering to lend funds to each borrower at a specified rate of interest. A borrower contracts with the lender who has the lowest interest rate offer. (If two lenders offer the same rate, the choice is arbitrary.) This process is repeated at the beginning of each period. Just before beginning the \(T\)-period game, sequentially rational borrowers infer the set of future interest rates that they will be offered given their knowledge of how lenders update their beliefs in a Bayesian fashion and the distribution of types in the industry. Effort choice functions for borrowers and Bayesian-updated probability assessments for lenders can then be defined for each possible credit history configuration in each period of the game. Internal consistency of the equilibrium requires that the interest rates that borrowers anticipate and condition their choice of effort on are such that, given their Bayesian-updated beliefs about borrowers, lenders just expect to earn their opportunity cost of funds from lending at such rates. Competition among lenders ensures that in equilibrium there is no incentive for either borrowers or lenders to diverge from these rates.

Before proceeding, some notation is required. Define \(h_t\) as a vector of length \(T\) where, for \(k \in \{1, \ldots, t - 1\}\), the \(k\)th element equals \(-1\) if the borrower defaulted in period \(t\), and \(1\) if the borrower made his payment in period \(t\), and for \(k \in \{t, \ldots, T\}\) the \(k\)th element equals \(0\). This vector completely describes the borrower's credit history as of the beginning of period \(t\). Now define \(H_t\) as the set of the \(2^T\) possible specifications of \(h_t\). Finally, define \(I^t_t(\pi_t)\) as a null vector of length \(T\) with a \(-1\) \((1)\) replacing the \(i\)th zero. Hence, a borrower with credit history \(h_t\), as of the beginning of period \(t\), will have a history \(h_t + I^t_t\) at the beginning of period \(t + 1\) if he defaults in period \(t\) and a history \(h_t + I^t_t\) if he makes the scheduled payment in period \(t\).

Let \(r_i(h_t)\) denote the interest rate that a borrower with credit history \(h_t\) anticipates in period \(t\). The set of all possible rates for period \(t\) is denoted \(r_i(H_t)\); that is, \(r_i(H_t) = \{r_i(h_t) : \forall h_t \in H_t\}\). Now define

\[
R = \{r_1(H_1), r_2(H_2), \ldots, r_T(H_T)\}.
\]

The set \(R\) specifies the anticipated interest rates for all possible credit histories and for all \(T\) periods. All borrowers face the same \(R\) because loan contracts can be conditioned only on the credit history. Each borrower therefore conditions the effort he plans to exert for each possible credit history of \(R\).

**Conditional effort function**

The conditional effort function, denoted \(e(t, R, h_t; \theta)\), specifies the level of effort that a type \(\theta\) borrower with credit history \(h_t\) will exert during period \(t\) taking the schedule of past, present and future anticipated interest rates, \(R\), as given. To characterize this function, let \(U(e, t, R, h_t; \theta)\) denote a type \(\theta\) borrower's discounted stream of expected utility from time \(t\) to \(T\), given that he has credit history \(h_t\), anticipates interest rates according to the schedule \(R\) and exerts effort \(e_t\) during period \(t\) and the optimal effort level thereafter. Also define \(V(t, R, h_t; \theta) = U[e(t, R, h_t; \theta), t, R, h_t; \theta]\). The conditional effort function and
the corresponding indirect expected utility function is defined by the following equation:

\[
V(t, R, h_t; \theta) = \max_{e_t \in [0, a]} \left\{ \int_{a^*(e_r, r_i(h_r))}^{\infty} \left[ \frac{\pi(e_r, a_r) - (1 + r_i(h_i))}{\omega^*(e_r, r_i(h_r))} \right] dF(\omega) - c(e_r; \theta) \right. \\
+ \delta[(1 - F[\omega^*(e_r, r_i(h_r))])V(t + 1, R, h_{t+1}; \theta)] \\
\left. + F[\omega^*(e_r, r_i(h_r))]V(t + 1, R, h_{t+1}; \theta) \right\}
\]

(1)

where $\delta$ is the borrower's discount factor; $0 < \delta < 1$.

Equation (1) indicates that a type $\theta$ borrower chooses his effort level in period $t$ (taking the interest rate schedule as given) in order to maximize the discounted stream of his expected utility from period $t$ through $T$. When choosing effort, the borrower takes into account the limited liability nature of the debt contract and the fact that his choice of effort in period $t$ affects the probability of his defaulting in period $t$, which in turn affects his expected future credit history and thus his expected utility for periods $t + 1$ through $T$.

Assuming an interior solution (i.e. the optimal $e_t$ satisfies $0 < e_t < \bar{e}$), the first-order condition corresponding to (1) can be written as

\[
\frac{\partial U(e_t, t, R, h_t; \theta)}{\partial e_t} = \int_{a^*(e_r, r_i(h_r))}^{\infty} \pi(e_r, a_r) dF(\omega) - c(e_r; \theta) \\
- \delta[V(t + 1, R, h_{t+1}; \theta)] \\
- V(t + 1, R, h_{t+1}; \theta)F'(\omega^*) \frac{\partial \omega^*}{\partial e_t} \\
= 0
\]

(2)

Assuming that the second-order conditions for (1) hold, the solution to (2) defines the conditional effort function $e(t, R, h_t; \theta)$. Its properties are examined in greater detail below.

**Lender's beliefs**

Sequential rationality requires that at the beginning of the game the lender's beliefs about the type of borrower she is dealing with when a loan offer is made must be specified at every possible information structure in the game. Let $P(\theta; t, R, h_t)$ denote the probability that the lender assigns to the event that she is dealing with a type $\theta$ borrower when, given $R$, a borrower with credit history $h_t$ is offered a loan in period $t$. By construction, $\sum_{\theta} P(\theta; t, R, h_t) = 1$. Because a rational lender will use Bayes’s rule to update her beliefs, it follows that $P(\theta; t, R, h_t)$ is defined recursively by

\[
P(\theta; t-1, R, h_{t-1}) \\
= \frac{[2 - j] + (-1)^{j}F(\omega^*[e(t-1, R, h_{t-1}; \theta), r_{t-1}(h_{t-1})])}{\sum_{k=1}^{\nu} P(\theta_k; t-1, R, h_{t-1})} \\
	imes [2 - j] + (-1)^{j}F(\omega^*[e(t-1, R, h_{t-1}; \theta_k), r_{t-1}(h_{t-1})])
\]

(3)

ened by:

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where \( i \) and \( j \) are indicator variables (used to reduce the amount of notation). In particular, \( i = p \) and \( j = 1 \) \((i = d \text{ and } j = 2)\) when default does not occur (default does occur) in period \( t - 1 \). Because a lender will use the actual distribution of borrowers in the industry, \( g(\theta) \), as her initial prior, it follows that \( P(\theta; 1, R, h_i) = g(\theta) \).

Equation (3) indicates that in period \( t \) the probability that a lender assigns to the event that she is dealing with a type \( \theta \) borrower given that she observes a credit history \( h_{t-1} + I^0_{t-1} \) \((h_{t-1} + I^d_{t-1})\) is equal to the corresponding probability in period \( t - 1 \) multiplied by the likelihood of no default (default) in period \( t - 1 \) given that the borrower was type \( \theta \), divided by the appropriate normalization factor. All lenders update their beliefs using (3) because all have access to the same information.

**Anticipated interest rates**

Sequential rationality requires that lenders must behave optimally with respect to the lowest interest rate that they would offer to a particular borrower. Optimality requires that a lender make this decision using her rationally formed beliefs about a borrower's type and the conditional effort function specified by each borrower's strategy. Let \( W(e, r) \) denote the lender's expected return (including original principal) from lending to a borrower who exerts effort \( e \) and agrees to pay interest at rate \( r \). If this borrower was type \( \theta \), the lender's expected return is

\[
W[e(t, R, h; \theta), r] = \int_0^{\omega(t, R, h; \theta, r(h))} \pi[e(t, R, h; \theta), \omega] \, dF(\omega) + [1 - F(\omega[t(e(t, R, h; \theta), r(h))])] [1 + r].
\]

Because the lender does not know the type of borrower she is dealing with, she averages her expected return over all types where the relative weights depend on her beliefs. Let \( r^*(h_i, R) \) denote the minimum interest rate that a lender will offer to a borrower with credit history \( h_i \) who anticipates rates \( R \). Assuming that all borrowers actually choose to borrow for all possible credit histories, \( r^*(h_i, R) \) can be derived from the solution to the following equation:

\[
\sum_{i=1}^{n} W[e(t, R, h_i \theta), r_i] P(\theta; t, R, h_i) = 1 + \rho.
\]

It is assumed that \( r^*(h_i, R) \) exists and is unique for each possible credit history. The schedule of all reservation interest rates is given by \( R^*(R) = \{R^*(R), \ldots, R^*(R)\} \), where \( R^*(R) = \{r^*(h_i, R) \forall h_i \in H\} \).

**Sequential equilibrium**

Each lender is willing to lend funds at the rates given by the schedule \( R^*(R) \) because at these rates lenders expect to earn \( 1 + \rho \). Lenders will therefore never offer a rate lower than that specified by \( R^*(R) \) because they will expect a loss at such rates. Borrowers will never accept an interest rate higher than that specified by \( R^*(R) \) because competition among lenders ensures that another
lender will offer a more favourable rate. Hence \( R^* = R^*(R^*) \) is the set of internally consistent equilibrium rates of interest that borrowers anticipate and lenders actually offer. It is assumed that this set of interest rates actually exists and is unique.

Equations (1), (3) and (5), together with the equilibrium condition \( R = R^* \), describe the equilibrium of the reputation game. Equation (1) describes each borrower's strategy with respect to conditional effort choice. Equation (3) specifies a lender's beliefs at all possible information structures. Using (5), a lender combines her beliefs with the conditional effort function of borrowers to derive her reservation interest rate schedule \( R^* \), which is also the equilibrium interest rate schedule.

II. IDENTIFYING THE REPUTATION EFFECTS

This section begins by formally defining a reputation effect.

**Definition 1.** For a given set of lender beliefs (i.e. a given probability assessment over the distribution of borrower types), reputation-induced effort in period \( t \) is the borrower's marginal response to the higher discounted expected utility from period \( t+1 \) to \( T \) that is available to the borrower if no default rather than default occurs in period \( t \). The reputation-induced decrease in borrowing costs reflects the lower risk brought about by the reputation-induced effort. Reputation-induced welfare is the additional welfare earned by the borrower that is attributable to the reputation-induced effort and lower cost of capital.

Reputation effects arise endogenously in the model and are easily identified in (2). Notice that, because a borrower's cost-of-capital period \( t+1 \) is conditioned on his credit history, he expects a different level of welfare as of period \( t+1 \), depending on whether he pays or defaults in period \( t \). Hence the borrower's marginal effort in period \( t \) has value stemming from two sources. First, as reflected by the first term in (2), it increases his expected returns in the current period. Second, as reflected by the last set of terms in (2), it lowers the probability of default in the current period (i.e. \( F'(x^*) \delta \omega^*/\delta e_0 \)) and hence raises the likelihood that the borrower will receive the utility stream in period \( t+1 \) associated with payment, \( V(t+1, R^*, h_i + I^*_e; \theta) \), rather than the utility stream associated with default, \( V(t+1, R^*, h_i + I^*_d; \theta) \). It is the additional effort associated with this latter incentive, and the corresponding change in the cost of capital and expected utility, that was defined above as the reputation effect.

There are now two questions that need answering. First, why does a borrower receive a higher expected utility stream in period \( t+1 \) if he makes his payment in period \( t \) rather than defaulting? Second, why are borrowers better off because of these reputation effects? The answer to the first question can best be explained in light of the following results (the proofs of which are contained in the Appendix).

**Lemma 1.**

\[
e(t, R^*, h_i; \theta_i) > e(t, R^*, h_j; \theta_i)
\]

\( \forall h_i \in H_i, \quad i > j \text{ and } t = 1, \ldots, T. \)
Lemma 2. Let \( D(t, h_{t-1}) \) equal the difference in the lender's estimate of a borrower's type in period \( t \) with and without payment in period \( t-1 \), assuming that the borrower's credit history at the beginning of period \( t-1 \) is \( h_{t-1} \).

\[
D(t, h_{t-1}) = \sum_{i=1}^{n} \theta_i P(\theta_i; t, R^*, h_{t-1} + P_{t-1}^d) - \sum_{i=1}^{n} \theta_i P(\theta_i; t, R^*, h_{t-1} + I_{t-1}^d) > 0
\]

\( \forall h_{t-1} \in H_{t-1} \) and \( t = \{2, \ldots, T\} \).

Lemma 3.

\( r^*_i(h_{t-1} + P_{t-1}^d) < r^*_i(h_{t-1} + I_{t-1}^d) \) \( \forall h_{t-1} \in H_{t-1} \) and \( t = \{2, \ldots, T\} \).

Lemma 4.

\( V(t, R^*, h_{t-1} + P_{t-1}^d; \theta) > V(t, R^*, h_{t-1} + I_{t-1}^d; \theta) \)

\( \forall h_t \in H_t \) and \( t = \{2, \ldots, T\} \).

Lemma 1 establishes that, when charged the same rate of interest, high-quality borrowers exert more effort than low-quality borrowers because the former have a lower disutility of effort than the latter. This result provides the base for Lemma 2, which states that lenders upgrade (downgrade) their assessment of the type of borrower they are dealing with when they observe a payment (default) the previous period. The logic is that high-quality borrowers are less likely to default and hence it is more likely that the borrower is higher- (lower)-quality than originally believed when a payment (default) is observed. Building on these two results, Lemma 3 indicates that lenders are willing to accept a lower cost of capital from borrowers who made a payment the previous period. The reason is that high-quality borrowers are more likely to have paid in the previous period and are also more likely to make payment in the current period. Finally, Lemma 4 indicates that a lower cost of capital in period \( t \), which stems from making payment rather than defaulting in period \( t-1 \), translates into a higher expected utility for the borrower from period \( t \) forward.

A central result of this section (a result already explicitly stated in Definition 1) is that reputation-induced effort is positive for all borrowers in all time periods (except the last). This result is stated more formally as

**Proposition 1.** A particular type of borrower facing a particular cost of capital exerts comparatively more effort if the period under consideration is not the last. Alternatively stated, a type \( \theta \) borrower facing a cost of capital \( r \) exerts comparatively more effort in period \( t \) if \( \delta \) (i.e. the discount rate) >0 rather than if \( \delta = 0 \).

Given Lemma 4, Proposition 1 follows directly from (2). The intuition underlying Proposition 1 has been discussed above but is worth repeating. A borrower exerts 'extra' effort in period \( t \) to lower the probability of defaulting during that period because making payment results in a higher stream of expected utility from period \( t+1 \) onward.

The extra effort exerted by the borrower is welfare-improving because without it the chosen level of effort is suboptimally low and as a result the
borrower's cost of capital is suboptimally high. In other words, reputation effects serve to offset at least some of the efficiency loss arising from moral hazard by implicitly allowing borrowers to commit to a higher level of effort, which in turn provides borrowers with a more efficient cost of capital. To see why effort without reputation effects is suboptimally low, solve (1) choosing both \( e \) and \( r_i \), which is equivalent to solving the first-best perfect-information problem. The first-order condition for this problem can be written as

\[
\int_0^1 \pi_i(e, \omega) \, d\xi - c_i(e_r; \theta) = 0.
\]

Noting that \( \omega^*(\cdot) > 0 \), a comparison of (2) and (6) reveals that the expected marginal value of effort (and hence the level of chosen effort) is higher in the first-best case than in the moral hazard/adverse selection case without reputation effects. The reason is that in the first-best case the borrower's cost of capital is contingent on the level of effort he exerts. Hence, higher effort translates into a lower cost of capital for the borrower because higher effort implies higher revenues for the lender when default occurs. With moral hazard, the borrower's cost of capital cannot be made contingent on effort. As a result, the marginal value of the borrower's effort is zero in default states because the higher revenues accruing to the lender in default states from extra borrower effort cannot be passed back to the borrower in the form of a lower interest rate.

An example

There are two types of borrowers: \( \theta_L \) ('low-quality') and \( \theta_H \) ('high-quality'), where \( \theta_L < \theta_H \). The fraction of low-quality borrowers is \( a \). The revenue function for both types of borrowers is given by

\[
\text{Revenue} = \begin{cases} 
\pi & \text{with probability } e \\
0 & \text{with probability } 1 - e,
\end{cases}
\]

where \( e \in [0, 1] \) denotes the borrower's choice of effort. The contracted principal with interest payment is denoted \( r \) and the disutility of effort function for a type \( \theta \) borrower is given by \( \theta e^2 / c \). The analogue of (1) can now be written as

\[
V(t, R, h_i; \theta) = \max_{e \in [0, 1]} \left\{ e_i[\pi - r_i(h_i)] - \frac{c}{\theta} e_i^2 + \delta[e_i V(t+1, R, h_i + I^e_r; \theta)] + (1 - e_i) V(t+1, R, h_i + I^d_r; \theta) \right\}
\]

The conditional effort function derived from (8) can be substituted for \( F(\omega^*[e(t, R, h_i, U)], r_i(h_i))] \) in (3) to specify the lender's beliefs because in this example \( e \) is probability of payment. The effort function and the lender's beliefs function can then be substituted into (5) to obtain the equation that defines
the equilibrium rate of interest:

\[ P(\theta_L; t, R, h_i)e(t, R, h_i; \theta_L) + P(\theta_H; t, R, h_i)e(t, R, h_i; \theta_H) \Delta \delta(h_i, R) = 1 + \rho. \]

The problem was solved using an iterative algorithm for \( T = 8 \) assuming that \( \pi = 1, c = 0.9, \rho = 0.1, \theta_L = 0.5555, \theta_H = 0.7777, \alpha = 0.5 \) and \( \delta = 0.95. \) By the beginning of period 8, the probability of being a low-quality borrower assigned by lenders ranged from 0.0831 for a borrower with a perfect credit history to 0.9855 for a borrower with seven consecutive defaults. In the first period of the unrestricted reputation game, \( e = 0.5540, \tau = 0.1490 \) and first-period welfare equals 0.2228 for the low-quality borrower. The corresponding values for the high-quality borrower are 0.7887, 0.1490 and 0.3112.

If lenders are restricted from conditioning loan contracts on a borrower's credit history (i.e., if reputation effects are suppressed), average effort in the first period decreases from 0.6714 to 0.6215 (approximately 7.5%). The reduced effort results in an increase in the first-period cost of capital from 0.1490 to 0.1609 (approximately 8%) and a decrease in average first-period welfare for borrowers from 0.2670 to 0.2608 (approximately 2%). In this particular example, average first-best welfare equals 0.2703, hence about 65% of the first-period welfare loss from moral hazard was eliminated by the reputation effects.

### III. DIMINISHING REPUTATION EFFECTS

In this section it is argued that reputation effects diminish over time as credit histories lengthen and adverse selection gradually disappears. Adverse selection gradually disappears and will eventually vanish (in the limit) as credit histories lengthen because lenders become increasingly informed over time about the type of borrower with whom they are dealing. Underlying this argument is the observation that reputation effects are non-existent without adverse selection, where a situation without adverse selection is one in which \( P(\theta_i; R^*, h_i) = \{0, 1\} \forall h_i \in H_i, t = \{1, \ldots, T\} \) and \( t = \{1, \ldots, n\}. \) That is, for a given credit history, lenders assign a probability of 1 that the borrower they are dealing with is of a particular type and a probability of zero that the borrower is of any other type.

**Proposition 2.** Without adverse selection, reputation effects are non-existent.

The proof and intuition underlying Proposition 1 is straightforward. If a lender knows the type of the borrower with certainty, then she also knows the exact incentives facing the borrower with certainty. Hence she is not prepared to offer a borrower with a better record of repayment a lower cost of capital, because the future incentives facing a borrower are unrelated to his credit history. Because the future cost of capital is unrelated to credit histories, borrowers have no incentive to exert extra effort to decrease the probability of defaulting in the current period. Therefore, reputation effects are non-existent.

If reputation effects are present with adverse selection but do not exist without adverse selection, then they must necessarily diminish over time as adverse selection gradually diminishes over time. The logic is that, as a lender collects an increasing amount of information about borrowers in the form of
longer credit histories, her distribution of beliefs will become increasingly precise for each borrower she interacts with. Hence, new information in the form of whether the borrower defaulted or paid in the previous period will cause less of a shift in this distribution. 

A smaller shift in the distribution implies that the cost-of-capital differential is smaller for a borrower who did (rather than did not) make his payment the previous period. The smaller cost-of-capital differential implies a lower future marginal benefit from paying rather than defaulting, and this induces the borrower to exert less effort, which in turn implies relatively lower reputation effects.

III. Credit Bureau Policy

If reputation effects diminish as credit histories lengthen, then aggregate borrower welfare may also decrease over time. Aggregate borrower welfare does not necessarily decrease as reputation effects diminish because reduced selection also has a direct effect on equilibrium welfare. This is because as adverse selection diminishes there is a shift from all borrowers being charged the same rate of interest to borrowers being charged individual rates that reflect the true risk of their investment. Vercammen (1994) examined these direct effects in a single-period version of this model and found that, because of the second-best nature of the equilibrium, their effect on aggregate welfare is generally ambiguous. This paper does not deal with the issue of comparing reputation effects with these direct effects. Rather, it is assumed that, at least over some range of the time horizon, diminishing reputation effects result in a decrease in aggregate borrower welfare.

The problem of specific interest is how reputation effects can be sustained over time to prevent a continuous decline in aggregate borrower welfare. In Holmstrom's model of reputation effects in labour markets, the specification of stochastic types is sufficient to sustain reputation effects. In this paper, it is proposed that credit market regulation can work to sustain such effects.

Before proceeding, it is important to ask whether the market itself is capable of sustaining reputation effects. In the case of single-period contracting, the answer is likely not. The reason is that a lender who tries to ignore information embedded in a borrower’s credit history in an attempt to maintain some adverse selection, and hence maintain reputation effects, will generally lose as customers all borrowers with highly favourable credit histories. This is because another lender who does not ignore information will identify the high-quality borrowers and offer them a relatively lower cost of capital because they are of relatively lower risk. Thus, if the original lender ignores information to try to maintain reputation effects, she will end up with a higher concentration of low-quality borrowers, and this will certainly discourage her from following such a practice.

In most industrialized countries, governments have implemented mechanisms for restricting the flow of information from borrowers to lenders. Most commonly, credit bureaus are restricted from releasing information about a borrower that is more than n periods old. While these policies perhaps make sense from a 'fairness' point of view, little has been written about their efficiency properties. In this section, such policies are shown to be useful in terms of maintaining a certain degree of adverse selection in the credit market and thus in sustaining reputation effects.
What are the determinants of the optimal number of periods to include in the floating credit history? Clearly, an excessively long credit history may not be desirable for the reasons discussed above. In this case, too much information is revealed to lenders, resulting in relatively weak reputation effects. However, an excessively short credit history is also not desirable. Indeed, it was shown that in the extreme case, where lenders were given zero access to credit history information, reputation effects did not exist. In general, for short credit histories, the benefit from exerting extra effort to decrease the probability of default is relatively low because evidence of this additional effort in the form of a superior credit history is too quickly erased. In other words, restrictive credit histories do not give borrowers sufficient opportunity to reveal themselves, and thus the incentive for borrowers to exert extra effort to achieve a more favourable credit history is relatively weak. The optimal credit history restriction trades off the negative incentive effects associated with long and short credit history restrictions.

The choice of the optimal credit history restrictions will also have distributional impacts. High-quality borrowers prefer relatively long histories in order to reveal their type to lenders and receive a more favourable cost of capital. On the other hand, low-quality borrowers prefer relatively short histories in order to obtain a more 'pooled' cost of capital that reflects the lender's uncertainty regarding the type of borrower she is dealing with. The optimal credit history restriction should therefore also depend on the relative weight that low- and high-quality borrowers receive from the social planner. Rather than examining the concepts discussed above formally, the notion of an optimal credit history restriction is illustrated with a simple example.

An example

Because this paper is not interested in diminishing reputation effects arising from a finite time horizon, it is necessary to examine the steady-state properties of an infinite-horizon problem. Unfortunately, solving for the steady state of an infinite horizon problem is computationally difficult, given the current structure of the model. However, the solution can be approximated by replacing detailed credit histories with the pair \( \{i, n\} \), where \( n \) is the number of periods that lenders are allowed to look back in a borrower's credit record and \( i \) is the number of defaults that the borrower has had in the previous \( n \) periods. Our solution is exactly correct if credit bureau policy is such that lenders only have information in the form \( \{i, n\} \) rather than the precise repayment history.

To modify (8), first note that a borrower with \( i \) defaults as of period \( t \) may end up with \( i - 1 \), \( i \) or \( i + 1 \) defaults as of period \( t + 1 \), depending on whether payment was made in period \( t \) and whether the period that gets dropped from the floating history (i.e. period \( t - n \)) was a default or payment period. There are \( i/n \) different credit history specifications that begin with a default. Thus, on average, a borrower with \( i \) defaults as of period \( t \) and who pays in period \( t \) will have \( i - 1 \) defaults the following period \((i/n) \times 100\%\) of the time and \( i \) defaults the following period \((1 - i/n) \times 100\%\) of the time. If the borrower had defaulted rather than paid in period \( t \), the corresponding numbers would be \( i \) and \( i + 1 \). Because borrowers should 'expect' to move according to these percentages, (8) is rewritten as

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(10) \[ V(t, R, i, n; \theta) = \max_{e, \epsilon \in [0, 1]} \left\{ e_r (\pi - r, i, n) - \frac{c}{\epsilon} \right\} \]
\[ + \delta \left( e_r \left[ \frac{i}{n} V(t+1, R, i-1, n; \theta) + \left( 1 - \frac{i}{n} \right) V(t+1, R, i, n; \theta) \right] \right) \]
\[ + (1 - e_r) \left[ \frac{i}{n} V(t+1, R, i, n; \theta) + \left( 1 - \frac{i}{n} \right) V(t+1, R, i+1, n; \theta) \right] \right) \}

The lender's beliefs in period \( t \) can be approximated as follows. If a borrower with \( i \) defaults in \( n \) periods exerted a constant level of effort \( \hat{e} \) over all \( n \) periods of the credit history, then the probability that such a borrower could have obtained such a history is \( \hat{\theta}^n \cdot (1 - \hat{\theta}) \). A lender could then use Bayes's rule to compute her posterior beliefs as

(11) \[ P(\hat{\theta}; t, R, i, n) = \frac{\alpha \hat{e}_r^{n-1} (1 - \hat{\theta})^i}{\alpha \hat{e}_r^{n-1} (1 - \hat{\theta})^i + (1 - \alpha) \hat{e}_r^{n-1} (1 - \hat{\theta})^i} \]

where the subscripts to \( \hat{e} \) refer to the type of borrower. In the above model, effort will not be perfectly constant over the \( n \) periods of the credit history. Nevertheless, the analysis proceeds as if it is in order that (10) can be used as a representation of the lender's beliefs. In the numerical solution, the arithmetic mean of effort over the \( n \) years of the credit history is used as a proxy for \( \hat{e} \).

Equations (9), (10) and (11), together with the steady-state condition \( V(t, R^*, i, n; \theta) = V(t+1, R^*, i, n; \theta) \), were solved to obtain the equilibrium under alternative credit history restrictions (i.e. the alternative values of \( n \)).

**Table 1**

**Effects of Alternative Credit Bureau Restrictions on Steady-State Beliefs and Welfare**

<table>
<thead>
<tr>
<th>Probability assessment that borrowers expect to be assigned by lenders</th>
<th>Credit bureau restrictions (( n )) (number of periods that lenders can look back in a borrower's credit record)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low type: PR('low')</td>
<td>2</td>
</tr>
<tr>
<td>0.582</td>
<td>0.624</td>
</tr>
<tr>
<td>High type: PR('high')</td>
<td>0.588</td>
</tr>
</tbody>
</table>

**Expected utility**

<table>
<thead>
<tr>
<th>Low type</th>
<th>High type</th>
<th>Both types</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.838</td>
<td>3.886</td>
<td>10.266</td>
</tr>
<tr>
<td>3.813</td>
<td>3.856</td>
<td>10.269</td>
</tr>
<tr>
<td>3.783</td>
<td>3.804</td>
<td>10.287</td>
</tr>
<tr>
<td>3.753</td>
<td>3.794</td>
<td>10.287</td>
</tr>
<tr>
<td>3.731</td>
<td>3.767</td>
<td>10.291</td>
</tr>
<tr>
<td>3.725</td>
<td>3.762</td>
<td>10.301</td>
</tr>
<tr>
<td>3.703</td>
<td>3.743</td>
<td>10.301</td>
</tr>
<tr>
<td>3.686</td>
<td>3.667</td>
<td>10.301</td>
</tr>
<tr>
<td>3.671</td>
<td>3.654</td>
<td>10.301</td>
</tr>
</tbody>
</table>

Table 1 shows the simulation results for \( n = 1, c = 0.9, \theta_L = 0.5, \theta_H = 0.8, \alpha = 0.5 \) and \( \delta = 0.95 \). Notice that as \( n \) increases the lender's steady-state probability assessment becomes increasingly precise (i.e. adverse selection diminishes). For example, with \( n = 2 \) a low-quality borrower expects that on average a lender will assign a 58.2% probability that he is of low-quality when he requests funds. This number rises to 84.7% for \( n = 10 \).

The level of steady-state welfare of the two types of borrowers is also presented as a function of \( n \). As discussed above, low-quality borrowers prefer highly restrictive histories and high-quality borrowers prefer relatively unrestrictive ones. If the welfare weights assigned to each type were specified, the
optimal credit history restriction could be identified. For example, if both types are weighted equally, \( n = 7 \) would be chosen as the optimal credit bureau policy because aggregate borrower welfare is at a maximum at this point.

V. Conclusions

The first objective of this paper was to show why reputation effects are likely to be important in credit markets characterized by moral hazard, adverse selection, and repeated interactions between borrowers and lenders. The second objective was to show that, because adverse selection is required for the existence of reputation effects and reputation effects enhance a borrower’s welfare, policies designed to preserve some level of asymmetric information may be warranted. The policy that was explicitly examined involved restricting credit bureaus from releasing information about a borrower’s past repayment record that was more than \( n \) periods old. This policy is not particularly efficient, because both excessively long and excessively short credit histories have negative consequences for reputation effects.

When viewed in the context of reputation effects, credit bureau restrictions have an intuitive appeal. If lenders are prevented from looking too far into a borrower’s past, borrowers continually have an incentive to forgo at least some of the short-run incentives to choose an excessively risky project or to reduce effort to a suboptimal level because there is always some scope for altering the lender’s beliefs. A single public firm supplying funds could simply ignore information if such action would raise the welfare of borrowers, and this would accomplish the same outcome as an explicit credit history restriction. Lenders in a competitive lending environment have no choice but to utilize all available information because of competitive pressures to offer to each borrower the rate that ensures an expected return equal to their opportunity cost of funds. Hence, intervention of the type discussed above may be warranted in a competitive credit market.

The model has a number of fairly restrictive assumptions that are sure to affect the applicability of our results. Unfortunately, owing to the complexity of the model, it is not immediately apparent how relaxing a particular assumption is likely to influence the relative importance of the reputation effects. For example, allowing for variable capital requirements will give rise to screening with either pooling or separating equilibria. Similarly, if lenders could easily collect information about the interest rates that a particular borrower paid in the past, screening over time would be possible. (For example, high-quality borrowers may accept rates higher than those prescribed by \( R^* \) to signal their type in order to secure a lower cost of capital in the future.) One could also allow borrowers to supply collateral. In this case, borrowers have relatively more to lose when default occurs. Finally, if the revenues of different borrowers were assumed to be correlated in a given time period, lenders would have a more complex information set when updating their beliefs.

APPENDIX

Proof of Lemma 1

If a type \( \theta_i \) and a type \( \theta_j \) borrower have the same credit history in period \( t \), then both types face the same pair of interest rates in period \( t+1 \). Hence, for equal amounts of
effort in period \( t \), both types expect the same revenues in period \( t \) and the same cost of capital in period \( t + 1 \). However, \( c_\theta(e; \theta_\gamma) < c(e; \theta_\gamma) \) and \( c_\theta > 0 \). The type \( \theta_\gamma \) borrower therefore has an incentive to exert less effort than the type \( \theta_\gamma \) borrower because both types want to equate discounted expected marginal revenue with their marginal disutility of effort (see (2)). Reducing effort decreases \( c(e; \theta_\gamma) \) and increases marginal expected revenues as given by the integrated expression in (2). Reducing effort also increases the expected future cost of capital for the type \( \theta_\gamma \) borrower, but the marginal loss associated with this increased cost is not offset by the marginal gain from reduced effort described above. If it were, the equilibrium would not be stable.

**Proof of Lemma 2**

To derive a more explicit expression for \( D(t, h_{t-1}) \) in Lemma 2, begin by substituting (3) into the expression for \( D(t, h_{t-1}) \):

\[
(A1) \quad D(t, h_{t-1}) = \sum_{i=1}^{n} \left[ \theta_i P(\theta_i; t-1, R^*, h_{t-1}) \left( \frac{-F^*(t-1, h_{t-1}; \theta_i)}{K_1} - \frac{F^*(t-1, h_{t-1}; \theta_i)}{K_2} \right) \right],
\]

where

\[
F^*(t, h_\theta; \theta) = F\left[ e^* \left( e(t, t^*, h_\theta; \theta), r(h_\theta) \right) \right];
\]

and

\[
K_j = \sum_{i=1}^{n} \left[ P(\theta_i; t-1, R^*, h_{t-1})(2-j) + (-1)^j F^*(t-1, h_{t-1}; \theta_i) \right], \quad j = 1, 2.
\]

Rearranging (A1) and cancelling terms results in

\[
(A2) \quad D(t, h_{t-1}) = \frac{1}{K_1 K_2} \sum_{i=1}^{n} \left\{ \theta_i P(\theta_i; t-1, R^*, h_{t-1}) \right. \\
+ \frac{1}{n} \left\{ \sum_{i=1}^{n} \left[ P(\theta_i; t-1, R^*, h_{t-1}) F^*(t-1, h_{t-1}; \theta_i) \right] \right. \\
- \sum_{i=1}^{n} \left[ P(\theta_i; t-1, R^*, h_{t-1}) F^*(t-1, h_{t-1}; \theta_i) \right] \right\}.
\]

Because the last summation term in (A2) equals 1 by definition, (A2) can alternatively be written as

\[
(A3) \quad D(t, h_{t-1}) = \frac{1}{K_1 K_2} \left\{ E_{\theta_{t-1}}[F^*(t-1, h_{t-1}; \theta)] - E_{\theta_{t-1}}[\theta F^*(t-1, h_{t-1}; \theta)] \right\},
\]

where \( E_{\theta} \) denotes the expectation operator with respect to \( \theta \) conditioned on the information available at time \( t \).

To sign \( D(t, h_{t-1}) \), note that \( K_1 \) and \( K_2 \) are both positive and that the numerator of (A3) is an expression for the negative of the covariance between \( \theta \) and \( F^*(t-1, h_{t-1}; \theta) \). Because \( F^*(e^*) > 0 \) and \( e^* = (\pi_e / \pi_{ew}) < 0 \), \( D(t, h_{t-1}) > 0 \) if and only if \( e(t, R, \theta) > e(t, R, \theta) \) for \( \theta > \theta_\gamma \). This last inequality holds by Lemma 1.

**Proof of Lemma 3**

Recall that \( W(e, r) \) is the lender's expected repayment when lending to a borrower who exerts effort \( e \) and agrees to pay interest at rate \( r \). Let \( W^\star[\theta; r^\star(h_\theta), h_\theta] = W[e(t, R, h_\theta; \theta), r^\star(h_\theta)] \). Using (4), it is straightforward to show that \( W_\theta = \int_{0}^{\infty} \pi_e dF(\omega) > 0 \). Because \( W_\theta > 0 \) and \( e(t, R^*, h_\theta; \theta) > e(t, R^*, h_\theta; \theta) \) for \( i < j \) by Lemma 1, it follows that \( W^\star[\theta; r^\star(h_\theta), h_\theta] > W^\star[\theta; r^\star(h_\theta), h_\theta] \) for \( i < j \). Now define

\[
(A4) \quad G(t, h_{t-1}) = \sum_{i=1}^{n} W^\star[\theta_i; r^\star(h_{t-1}), h_{t-1} + I_{t-1}^r] P(\theta_i; t, R, h_{t-1} + I_{t-1}^r) - \frac{1}{n} \sum_{i=1}^{n} W^\star(\theta_i; r^\star(h_{t-1}), h_{t-1} + I_{t-1}^r) P(\theta_i; t, R, h_{t-1} + I_{t-1}^r).
\]

Using a proof similar to the one for Lemma 2, and making use of the fact that \( W^\star[\theta_i; r^\star(h_\theta), h_\theta] > W^\star[\theta_i; r^\star(h_\theta), h_\theta] \) for \( i > j \), it is straightforward to show that

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$G(t, h_{-1}) > 0$. That is, a lender expects a higher return from borrowers who made the payment the previous period than from those who never made a payment the previous period.

Because a lender expects a return of $\rho$ on all transactions, it follows that

$$
(A5) \quad \sum_{i=1}^{n} W^*(\theta \mid r_i^*(h_{-1} + I_{i-1}^d), h_{-1} + I_{i-1}^d)P(\theta \mid t, R^s, h_{-1} + I_{i-1}^d)
$$

$$
= \sum_{i=1}^{n} W^*(\theta \mid r_i^*(h_{-1} + I_{i-1}^d), h_{-1} + I_{i-1}^d)P(\theta \mid t, R^s, h_{-1} + I_{i-1}^d).
$$

If $(A5)$ is to be consistent with $G(t, h_{-1}) > 0$, it must be the case that $r_i^*(h_{-1} + I_{i-1}^d) < r_i^*(h_{-1} + I_{i-1}^d)$. The reason is that, if $r_i^*(h_{-1} + I_{i-1}^d) = r_i^*(h_{-1} + I_{i-1}^d)$, then the expression on the left-hand side of $(A5)$ minus the expression on the right-hand side equals $G(t, h_{-1})$. Because $G(t, h_{-1}) > 0$, the left-hand side of $(A5)$ exceeds the right-hand side when $r_i^*(h_{-1} + I_{i-1}^d) = r_i^*(h_{-1} + I_{i-1}^d)$. Equation $(A5)$ will therefore hold as an equality only if Lemma 3 is true because $W^*(\theta \mid r_i^*(h), h)$ is non-decreasing in the contracted interest rate at the equilibrium point.

**Proof of Lemma 4**

First we establish that a borrower's welfare in period $t$ is higher the lower the cost of capital in period $t$. This is accomplished by totally differentiating $(1)$ and then substituting in $(2)$ and the identity $\pi(e, w) = l + r$:

$$
(A6) \quad \frac{dV(t, R, h; \pi)}{dr} = -[1 - F(w^*[\pi])] < 0.
$$

Lemma 3 establishes that a borrower who does not default in period $t$ enjoys a lower cost of borrowing in period $t+1$. This result, combined with $(A6)$, implies that a borrower's welfare in period $t+1$ is higher if he makes his payment in period $t$ rather than defaulting.

**ACKNOWLEDGMENTS**

I am grateful to Brian Wright, Jeff Perloff, Michael Katz, workshop participants at the University of Arizona and two anonymous referees for helpful comments and suggestions. The usual disclaimer applies. Financial support from the Social Science and Humanities Research Council of Canada is acknowledged.

**NOTES**

1. In the finance literature, John and Nachman (1985) and Spatt (1983) examine the relationship between corporate insiders and external debt-holders and find that reputation acquisition can help alleviate the problem of insiders passing up projects that have a positive net present value. In the literature on sovereign lending, Eaton (1989) and Soares (1988) have constructed reputation models in an attempt to explain the existence of unsecured loan contracts.

2. The indivisible capital assumption implies that costs often associated with adverse selection, such as investment signalling (e.g. Mibe and Riley 1988, Innes 1991) or inefficient pooling investment choices (e.g. De Meza and Webb 1989), are not present. This suggests that the potential benefits of adverse selection vis-à-vis reputation effects may be overstated.

3. To keep the analysis focused on reputation effects, it is assumed that borrowers are not able to carry excess revenues into the next period (i.e. there is no self-financing). Diamond also employs this assumption.

4. A subscript on a function denotes a partial derivative.

5. Innes (1990) shows that standard debt contracts are the most efficient of all monotonic forms when moral hazard of the type described above exists. The reason is that borrowers have an incentive to reduce effort (relative to the full-information outcome) because the benefits from marginal effort are shared with outside investors. Standard debt contracts maximize a borrower's payoff in high profit states more than any other type of monotonic contract form because it gives him the greatest incentive to commit implicitly to an effort level that is as close to first-best as possible.

6. Technically, when $\phi < \phi^*(e, r)$, then $\omega^*(e, r)$ does not exist because the borrower's revenues will fall short of his debt obligations under all states of nature. However, $\omega^*(e, r) = 1$ ensures that
the borrower’s expected revenues after debt service equal zero, just as they do when \( w^*(e, r) \) does not exist, and thus is consistent with the mathematical formulation.


8. Specifically, the term in square brackets in (2) is positive by Lemma 4, \( F'(w^*) > 0 \) and \( w^* > 0 \). Hence the expected marginal value of effort is higher with \( t < T \) than with \( t = T \) and with \( \delta > 0 \) than with \( \delta = 0 \).

9. Details of the solution procedure are available from the author upon request.

10. The result that the precision of the posterior distribution increases, and that marginal sample information has less of an effect on the mean of the posterior distribution as the size of the sample increases, is well documented in Bayesian statistics.

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