Consumer Rationality and Credit Cards

Dagobert L. Brito

Rice University

Peter R. Hartley

Rice University and Australian National University

Borrowing on credit cards at high interest rates might appear irrational. However, even low transactions costs can make credit cards attractive relative to bank loans. Credit cards also provide liquidity services by allowing consumers to avoid some of the opportunity costs of holding money. The effect of alternative interest rates on the demand for card debits can explain why credit card interest rates only partially reflect changes in the cost of funds. Credit card interest rates that are inflexible relative to the cost of funds are not inconsistent with a competitive equilibrium that yields zero profits for the marginal entrant.

I. Introduction

Several authors, most notably Ausubel (1991), have argued that even though the market for bank credit cards is unregulated, has thousands of independent firms, many of them recent entrants, and has millions of consumers, it nevertheless appears to be noncompetitive. The apparent inflexibility of credit card interest rates relative to the cost of funds is the evidence most commonly cited in support of this claim.[1] Ausubel also argues that the prices paid by banks for credit card receivables indicate that banks earn above-normal profits on those assets. He attributes the failure of competition partly to the

We thank Robert E. Lucas, Jr. and two referees for valuable comments, and the Baker Institute at Rice University for financial support.

[1] Average credit card interest rates have fallen in recent quarters but by less than the cost of funds.

[*Journal of Political Economy*, 1995, vol. 103, no. 2]

failure of consumers to anticipate "the very high probability that they will pay interest on their outstanding balances" (p. 50).[2]

We show that rational individuals who face transaction costs in borrowing may finance a substantial fraction of their consumption with credit card debits on which they *expect* to pay interest. This will be true even when the interest rate on credit cards is substantially above the interest rate on alternative sources of funds. We show that credit cards are useful not only when the individual's transaction demands are uncertain but also when the individual is attempting to smooth a deterministic income or consumption stream. The latter result is interesting since credit cards are used to finance foreseeable periods of high expenditure such as Christmas and vacations.

Credit cards serve two basic purposes. On the one hand, they compete with precautionary money balances as a medium for financing transactions.[3] Most credit cards have a "grace period" whereby interest charges can be avoided by paying off the outstanding balance in full at the end of each month. Further, a consumer holding multiple credit cards can, by clever manipulation, borrow an amount equal to a month's transaction interest free as long as he or she wants.[4] Customers who can avoid interest charges might be expected to use their credit cards to finance all their expenditures. Presumably, they do not do so only because credit cards are unacceptable, or more inconvenient than cash or checks, for some transactions. We assume for most of the paper that interest starts accruing the moment charges are posted to the account. Consumers in our model will therefore be less inclined to use credit cards as a transactions mechanism than they would if they could use them without any explicit monetary charge. Our analysis shows, however, that rational consumers who *do* have to make quite high interest payments on credit card transactions would nevertheless continue to use their cards as a transactions mechanism.[5]

Credit cards also compete with bank loans and other forms of fi-

[2] He suggests instead that "many consumers may not understand how interest rates work and underestimate the consequences of borrowing" (p. 71, n. 42).

[3] The substitutability of credit cards for money balances raises interesting questions about the role of money and the conduct of monetary policy in an economy in which transactions can be financed by credit instruments. Black (1970), Hester (1972), and Fama (1985), among others, have discussed these issues.

[4] Hester (1972, p. 285) notes that "the charge card allows [its owner] to shift the burden of carrying zero-interest-bearing transactions balances from himself to those issuing the card for at least a month. With a charge card he can reduce his demand account balance[s] . . . [and] it is less important to waste time and effort trying to minimize them."

[5] Ausubel (1991, p. 70, table 11) reports that roughly 25 percent of customers avoid finance charges by always paying off their accounts in full each month. Our analysis focuses on the remaining 75 percent of customers who use their cards even though they incur interest charges.

nancing as a way of smoothing irregular consumption or income flows, or providing insurance against unanticipated shocks to expenditure or income. They allow individuals to borrow amounts within their credit limit at close to zero transaction costs. We show that very small transactions costs of arranging alternative low-interest financing can make it rational for consumers to borrow using their credit cards. A senior bank officer told us that the costs to the bank of processing a loan are so high that they cannot afford to make a loan of less than $3,000 for 1 year except at interest rates above those charged on credit cards. The implication that banks are not good alternatives to credit card debits for amounts less than $3,000 would be strengthened when we account for the transaction costs imposed on the borrower. It is unlikely that the sale of assets is a cheaper source of money for amounts under $3,000. For many individuals, credit cards have replaced finance companies, pawn shops, and layaway plans.[6]

Our model allows us to explain the insensitivity of credit card interest rates to changes in the cost of funds. The nature of consumer demand for credit card loans implies that card holding would be extended to customers having a higher risk of default following a drop in the cost of funds. The required spread between credit card loan rates and the cost of funds therefore rises as the cost of funds falls. The idea that inframarginal users of credit cards have a lower default risk also can explain the premium that appears to have been paid for some portfolios of credit card receivables.

II. Financial Structure

Consider first the transactions motive for using credit cards. We assume that consumers can make new investments or arrange loans only at discrete intervals.[7] In contrast, money balances or credit card debits are always available to finance purchases. We assume that the interest yield on new investments exceeds the interest yield on liquid assets, and the interest cost of new loans is below the interest cost of card debits. The continuous availability of money or credit card debits implies, however, that they yield liquidity services to offset the interest differentials.[8] We shall assume that the opportunity cost of money

[6] Inquiries in Houston in February 1992 revealed rates ranging from 17 percent and a $100 fixed fee for a collateralized 1-year loan at a branch of a major national finance company to over 50 percent for small loans ($300 maximum) at a local finance company.

[7] We shall take the spacing of these intervals to be a month when we are discussing the transactions demand for credit cards and cash. In a more general model, the period would be determined endogenously by transactions costs and interest yields.

[8] Lucas (1984) and Lucas and Stokey (1983, 1987) consider cash-in-advance models in which some goods can be purchased "on credit." These goods are paid for in capital

balances is approximated by the return on Treasury bills. The cost of using a credit card instead of money is that a higher interest rate must be paid at the end of each period on balances debited to the card within the period.

When we are analyzing the use of credit cards for borrowing, we shall think of bank loans with a horizon of about 1 year as the relevant alternative liability.[9] The real interest rate on these loans is likely to exceed the opportunity cost of money balances. In addition, borrowers from a bank are likely to incur costs of at least \$100 for each transaction.

Sections III, IV, and V of the paper analyze the problem of minimizing the cost, or the expected cost, of financing a given stream of expenditures.[10] The key result is that the demand for credit card financing depends primarily on the ratio of the interest rate on loans to the credit card interest rate. This result is crucial to our analysis of the market equilibrium. Section VI shows that utility maximization introduces additional effects due to risk aversion and endogenous variations in desired consumption in response to variations in the cost of financing expenditure. Nevertheless, the demand for credit card financing again depends on the ratio of the interest rate on loans to the credit card interest rate.

III. Smoothing Deterministic Income or Consumption Streams

Consider a consumer with wealth W at $t = 0$ who wishes to maximize wealth at $t = 1$. The consumer faces a *net* expenditure constraint, $c(t) > 0$ for $t \in [0, 1]$, that can be financed either by depleting money balances, $m(t) \geq 0$, which must be chosen at $t = 0$, or by borrowing against a credit card at an interest rate i. We denote the amount of borrowing on the credit card at time t by $d(t)$. In addition, at $t = 0$

market trading at the end of the period. To ensure a positive demand for money, they assume that a subset of goods cannot be purchased on credit. They justify the assumption by arguing that the "transactions costs" associated with making credit purchases exceed the transactions costs of making cash purchases. In our model, all goods can be purchased using either cash or credit. Cash will still be used, however, since the interest payable on credit card purchases exceeds the interest rate on assets available in capital market trading. Our assumption of a differential interest rate can be considered as an alternative way of representing transactions costs.

[9] Since the transactions costs of arranging a bank loan are independent of the period of the loan, credit cards become relatively more attractive the shorter the intended period of the loan. The assumption that the loan will last a full year probably also biases the analysis against the use of credit cards.

[10] This model is closely related to the classic analyses of the transactions demand for money by Baumol (1952) and Tobin (1956).

the consumer can invest an amount $l(0)$ in an asset that has a rate of return r.

Net wealth at $t = 1$ is given by

$$l(1) + m(1) - d(1). \tag{1}$$

This is to be maximized subject to an expenditure constraint

$$c(t) = \mu(t) + \delta(t) \quad \text{for } t \in [0, 1], \tag{2}$$

where $\mu(t)$ denotes the change in money balances at time t and $\delta(t)$ the change in credit card debits at time t. Money balances and credit card debits therefore evolve according to differential equations

$$\dot{m}(t) = -\mu(t) \tag{3}$$

and

$$\dot{d}(t) = \delta(t) + id(t). \tag{4}$$

In addition, investments in $l(t)$ accumulate at the rate r:

$$\dot{l}(t) = rl(t). \tag{5}$$

Choices at $t = 0$ also have to satisfy the initial wealth constraint

$$W = l(0) + m(0), \tag{6}$$

whereas cash balances can never become negative:

$$m(t) \geq 0. \tag{7}$$

Let $\lambda(t)$ be the multiplier associated with the expenditure constraint (2) and $\nu(t)$ the multiplier associated with the nonnegativity constraint (7). Let the costate variables for the differential constraints (3), (4), and (5) be $p(t)$, $q(t)$, and $s(t)$, respectively. Define the Hamiltonian as

$$H = -p\mu + q(\delta + id) + srl + \nu m + \lambda(\mu + \delta - c). \tag{8}$$

The first-order conditions for a maximum are

$$-p + \lambda \leq 0 \quad \text{with } \mu(-p + \lambda) = 0 \tag{9}$$

and

$$q + \lambda \leq 0 \quad \text{with } \delta(q + \lambda) = 0; \tag{10}$$

the expenditure constraint

$$\mu(t) + \delta(t) - c(t) = 0; \tag{11}$$

differential equations for the costate variables

$$\dot{p} = -\nu \quad \text{with } \nu m = 0, \tag{12}$$

$$\dot{q} = -iq, \tag{13}$$

and

$$\dot{s} = -rs; \tag{14}$$

and the transversality conditions

$$-q(1) = s(1) = 1. \tag{15}$$

Finally, since wealth can be reallocated between $l(0)$ and $m(0)$ at $t = 0$, we must also have

$$p(0) = s(0). \tag{16}$$

From (14) and (15), $s(0) = e^r s(1) = e^r$; thus from (16), $p(0) = e^r$. On the other hand, from (13) and (15), $-q(0) = e^i > p(0)$ for $i > r$. From the expenditure constraint (11), at least one of μ or δ must be strictly positive at each $t \in [0, 1]$. If $\delta(0) > 0$, then, from (9) and (10), $-q(0) = \lambda \leq p(0)$, which results in a contradiction. Hence, $\delta(0) = 0$ and $\mu(0) > 0$, from which we conclude that $m(0) > 0$, and at $t = 0$, consumers will use money balances to finance their consumption. Then for all t such that $m(t) > 0$, $\nu = 0$ and $\dot{p} = 0$, and as long as consumers hold positive money balances, $p(t)$ will not change. However, $-q(t)$ is declining monotonically at the rate i. The situation is represented in figure 1.

Let t_s be the first time in which $-q(t_s) = p(t_s)$. Then from $p(t_s) = s(0) = e^r$ and the differential equation (13), we deduce that

$$e^r = -q(t_s) = e^{i(1-t_s)}, \tag{17}$$

so that $t_s = 1 - (r/i) < 1$. If $m(t_s) > 0$, then from (12), $\nu(t_s) = 0$, $\dot{p}(t_s) = 0$, and $p(t)$ will remain constant. On the other hand, from (10), λ

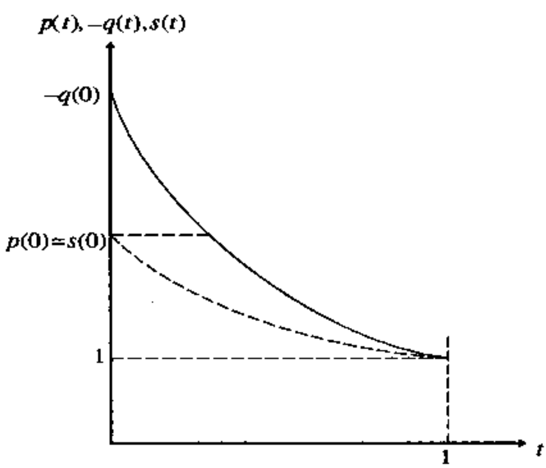

FIG. 1.—Time paths for the costate variables

$\leq -q(t)$, which is declining at the rate i. Thus $\lambda(t)$ is also declining, and if $p(t)$ is constant we must have $\lambda(t) < p(t)$ for $t > t_s$. Then, from (9), $\mu = 0$ and $m(t)$ and $p(t)$ will not change for $t > t_s$. But if $m(1) > 0$, money balances along with l and d could be reallocated at $t = 1$, and the transversality condition $p(1) = 1$ would apply. This, however, would contradict the conclusion that $p(t)$ remains constant at $p(0) > 1$. We conclude that $m(t) = 0$ for all $t \in [t_s, 1]$.

The consumer will consume from money balances for $t < t_s$ and finance purchases with a credit card for $t \geq t_s$.[11] Equation (17) therefore implies that consumption will be financed by credit card debits for a fraction r/i of the time and by money balances for a fraction $1 - (r/i)$ of the time. For a real interest rate r of about 0.35 percent per month (which compounds to an annual rate of about 4.2 percent) and a real interest rate i of about 1.5 percent per month (which compounds to an annual rate of about 19.6 percent), credit cards with interest payable on all debits made during the month would be used to finance about 23 percent of expenditures.

The solution above was derived on the assumption that $m(0) > 0$. Observe that as long as purchases are financed with a credit card, $\delta(t) > 0$ and $\lambda(t) = -q(t)$. However, $q(t)$ satisfies the differential equation (13) so that if purchases are financed only by credit card debits, then $\lambda(0) = e^i\lambda(1) = e^i$. On the other hand, the first-order condition (9) implies $\lambda \leq p$. Thus, if consumers do not hold money balances, $\lambda(0) \leq p(0) = e^r$, which results in a contradiction when $i > r$. When the interest rate on credit card debits exceeds the interest rate on alternative assets, some consumption will be financed with money balances. However, a fraction r/i will also be financed using credit cards.

This analysis would have to be modified if there is a grace period in the sense that interest is not payable on credit card balances that satisfy two conditions: first, consumers have sufficient liquid assets at the end of the billing cycle to pay for any purchases made during the cycle *and*, second, the outstanding balance at the beginning of the billing cycle was zero. If either of these two conditions fails to hold, the previous argument would remain valid since interest charges would start to accrue upon purchase. If these two conditions hold, however, equation (4) would be written as

$$\dot{d}(t) = \delta(t), \tag{18}$$

and equation (13) would now be

$$\dot{q} = 0. \tag{19}$$

[11] The result that previously chosen cash balances will be "used first" to finance consumption is related to the result in Hartley (1988) that cash will be used first to finance consumption in a model in which consumption can also be financed by interest-bearing demand deposits.

From (15) and (19), $-q(t) = 1$ for all $t \in [0, 1]$. As before, (14), (15), and (16) imply $p(0) = e^r > 1$. If $\mu(0) > 0$, then from (9) and (10), $p(0) = \lambda \leq -q(0)$, which results in a contradiction. Hence, $\mu(0) = 0$, $\delta(0) > 0$, and consumers will use credit cards to finance their consumption throughout the period. Throughout the remainder of the paper, we assume that there is no grace period. This will bias the analysis against the use of credit cards as a transactions device.

IV. Credit Cards as an Alternative to Bank Loans

Now suppose that consumers are contemplating financing net expenditure over the coming year. To simplify matters, assume that initial wealth is zero. The consumer can finance the expenditure with credit cards. Alternatively, after paying the fixed cost K, the consumer can borrow from a bank at an interest rate r and finance at least part of the expenditure with money balances. In the latter case, the maximization problem will be identical to the problem just analyzed, except for the presence of the fixed transactions cost. Also, since we are thinking of the alternative asset as a bank loan instead of money, the alternative real interest rate r will be the bank loan rate rather than the Treasury bill rate.

The cost of financing consumption through t_s by borrowing is

$$\left[\int_0^{t_s} c(t)\,dt + K\right]e^r. \tag{20}$$

Alternatively, the cost of borrowing that amount using credit cards is

$$\left[\int_0^{t_s} e^{-it}c(t)\,dt\right]e^i. \tag{21}$$

Beyond t_s, the two methods have the same cost since credit cards will be used in either case until $t = 1$, at which point both debts will be retired. The consumer will borrow at the rate r only if

$$K < e^{i-r}\int_0^{t_s} e^{-it}c(t)\,dt - \int_0^{t_s} c(t)\,dt. \tag{22}$$

For illustrative purposes, suppose that $c(t)$ is constant at $\bar{c}$. Then inequality (22) becomes

$$K < \bar{c}\left[e^{i-r}\left(\frac{1 - e^{-it_s}}{i}\right) - t_s\right]. \tag{23}$$

TABLE 1

Minimum Loan with K = $100

i	r = .07	r = .09
.15	$2,433.78	$3,267.00
.18	1,752.13	2,156.06
.21	1,362.69	1,600.67

TABLE 2

Credit Card Switching Costs

$i - \rho$	i = .18	i = .19	i = .20
.01	$5.62	$5.66	$5.70
.02	11.21	11.29	11.36
.03	16.76	16.87	16.99

Now use result (17) that $t_s = (i - r)/i$ and define $L = \bar{c}t_s$ to write (23) as

$$\frac{K}{L} < \frac{e^{i-r} - 1}{i - r} - 1. \tag{24}$$

Table 1 sets out the minimum values of L at which bank loans are competitive for various values of the annual real interest rates on bank loans, r, and credit cards, i, assuming K = $100. Even moderate transactions costs can lead to substantial borrowing on credit cards. Inequality (24) also implies that the derivative with respect to i of the minimum competitive bank loan is opposite in sign but equal in magnitude to the derivative with respect to r.

A similar calculation reveals the likely sensitivity of consumers to interest rate differentials between credit cards. Let r be the rate on an alternative credit card. From (21) and under the assumption that all expenditure in excess of income in the period is financed by credit cards, it is rational for the consumer to change credit cards only if

$$e^{i}\int_0^1 e^{-it}c(t)\,dt - e^{\rho}\int_0^1 e^{-\rho t}c(t)\,dt > K_c, \tag{25}$$

where K_c are the transaction costs associated with changing credit cards.

For a selection of values of i and $i - \rho$, table 2 gives the level of K_c sufficient to prevent a change in the credit card used to finance a planned expenditure of $1,000 evenly spread over a year. Thus, for example, an individual who planned to finance $1,000 in expenditure evenly spread over a year and whose credit card was charging 19 percent would not change to a credit card charging 18 percent if the cost of changing exceeded $5.66.

The numbers in table 2 suggest that a differential in the annual rates in the range of 3 percent is unlikely to cause consumers with average balances to change credit card companies. If consumers do not intend to maintain a positive balance on their card for the whole year, transactions costs much lower than those presented in table 2 would be sufficient to prevent a change in cards. For example, if the consumer expected to finance the $1,000 balance for only a month and the current *monthly* interest rate was 1.8 percent (23.87 percent annually), transactions costs of $1.52 would be sufficient to prevent a switch to a card with a monthly interest rate of 1.5 percent (19.56 percent annually).

We argue in Section VII that the transactions costs of switching cards play a crucial role in determining the nature of the equilibrium in the market for credit card loans. Since the thresholds for switching cards are linear in the average balance, individuals with a high balance are more likely to be attracted by a lower interest rate. People who intend to hold large credit card balances for some time, however, are more likely to be bad credit risks. Large borrowers who also have good credit ratings will most likely have access to cheaper sources of financing and would not rationally maintain large credit card balances. A bank that attempts to compete by offering lower interest rates may therefore suffer from adverse selection and would need to carefully screen customers wishing to transfer large balances from other cards.

Conversely, those customers who are more desirable will tend to respond more to increases in interest rates. The most desirable customers are those who borrow a substantial amount on their cards and yet remain well within their credit limits and therefore are unlikely to default. Any of these customers holding more than one card can switch their balances between cards at a relatively low transaction cost. They would respond to a rise in interest rates and, unlike the new customers attracted by lower rates, would tend to be good credit risks and therefore undesirable customers for the bank to lose.[12]

V. Consumption Insurance

In this section, we briefly examine the use of credit cards as a cost-minimizing form of insurance against unanticipated shocks to desired

[12] This adverse selection theory is similar to the one proposed by Ausubel (1991, p. 70), except that the switching cost theory does not rely on consumer irrationality.

consumption. We show that the ratio of the interest rate on loans to the interest rate on credit cards is also the primary determinant of the probability that consumers use their credit cards to finance purchases.

Let t and $t + 1$ denote two periods in which the consumer deals in the asset market. Suppose that desired consumption between these periods is randomly distributed with a probability distribution function $F(c)$. Since consumers can engage in capital market trades only at discrete intervals, money balances must be chosen before the actual level of consumption is known. The opportunity cost of a given level m of real money balances therefore is independent of the amount of money spent on consumption goods during the period.

On the other hand, consumers who finance consumption by credit card debits can alter the cost of financing their purchases by the actions they take within the period. Credit card debits incur an interest cost of i times the total debits outstanding at the end of the period. Consumers therefore have an incentive to minimize purchases made with credit cards. They will use any money balances they have on hand to finance consumption before they resort to credit card debits.

Let r denote the interest rate on alternative assets. The opportunity cost of holding real money balances of m for the period $[t, t + 1]$ will be rm. The expected cost of credit card debits over the period is

$$\int_m i(c - m)\, dF(c). \tag{26}$$

Total expected costs of financing consumption during the period are

$$rm + i \int_m (c - m)\, dF(c). \tag{27}$$

These costs will be minimized where

$$r = i \int_m dF(c) = i \Pr(c > m). \tag{28}$$

Equation (28) has a straightforward economic interpretation. The interest rate r reflects the opportunity cost of holding the marginal money balance. The *expected* marginal benefit of holding this additional unit of money is the saving in interest on a unit of credit card debits, i, times the probability that consumption will be large enough to require credit card financing.

Rearranging (28), we deduce that the probability of using credit cards in any given period will be r/i. In Section IV, we deduced that purchases would be financed by credit cards for a fraction r/i of the time. Equation (28) implies that, in an uncertain environment, consumption would be financed by credit cards for a fraction r/i of the possible states.

TABLE 3

REAL MONEY BALANCES AND AVERAGE CARD DEBITS

	r = .003		r = .0045	
i	m_0	$\bar{d}$	m_0	$\bar{d}$
	c Is Lognormal with Mean 1,000, Standard Deviation 1,500			
.012	\$1,153.69	\$371.08	\$783.97	\$484.47
.015	1,383.18	319.76	980.23	418.60
.018	1,585.56	282.82	1,153.69	371.08
	c Is Negative Exponential with Mean 1,000			
.012	1,386.29	250.00	980.83	375.00
.015	1,609.44	200.00	1,203.97	300.00
.018	1,791.76	166.67	1,386.29	250.00

If we denote the solution to (28) by m_0, then average credit card debits will be

$$\bar{d} = \int_{m_0} (c - m_0)\, dF(c). \tag{29}$$

Average credit card debits will therefore be higher the more the probability distribution $F(c)$ is skewed toward large values of c.

Table 3 gives the values for m_0 and $\bar{d}$ for alternative values of r and i and for $F(c)$ given by a lognormal distribution and a negative exponential distribution. The real interest rates r and i have been set to representative *monthly* real rates on Treasury bills and credit card balances. A monthly interest rate of 0.3 percent compounds to an annual rate just under 3.66 percent, whereas a monthly rate of 1.8 percent compounds to an annual rate just over 23.87 percent. Even for such a large discrepancy in interest rates, average monthly credit card debits are close to \$280 in the lognormal case and \$166 in the negative exponential case.[13]

We are also interested in the effects of changes in i and r on the demand for the transactions services of credit cards. If we let $f(c)$ be the probability density function corresponding to the distribution function $F(c)$, then by differentiating (28) and (29) we can easily show

[13] The numbers in table 3 may understate the benefits of borrowing on credit cards. The assumption that individuals can borrow as well as lend at the low alternative rate r will not be valid for most consumers. Also, the effective interest rate on credit card debits could be lower than the contracted rate for individuals who pay back their loans soon after the grace period.

that

$$\frac{d\bar{d}}{dr} = \int_{m_0} \frac{1}{if(m_0)}\,dF(c),$$
$$\frac{d\bar{d}}{di} = -\frac{r}{i}\int_{m_0} \frac{1}{if(m_0)}\,dF(c). \tag{30}$$

Therefore, since $r < i$, average credit card debits $\bar{d}$ respond more to changes in the opportunity cost of holding money balances than to the interest rate on credit cards.

VI. Utility Maximization

We now assume that consumers maximize a time additively separable expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \theta_t) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \theta_t \frac{c_t^{1-\gamma}}{1-\gamma}, \tag{31}$$

where $0 < \beta < 1$ is the time rate of discount, γ is the coefficient of relative risk aversion, c_t is consumption during period t, and θ_t is a shock to desired consumption within period t. For analytical convenience, we assume that θ_t is uniformly distributed over the interval $[\Theta_1, \Theta_2]$ and θ_t and θ_s are independent for $t \neq s$. We shall use $\Theta_R = \Theta_2 - \Theta_1$ to denote the range of θ. *The key result in this section is that utility-maximizing consumers will choose money balances and credit card debits so as to set the ratio of the expected liquidity value of cash balances to the expected liquidity value of credit card debits equal to* r/i.

We again assume that money balances that yield no interest income can be used to finance consumption at any time. Consumers can also hold loans, yielding a constant positive rate of return r each period, that *cannot* be used to finance purchases during period t. They can also borrow from the capital market at the beginning of period t.[14] For simplicity, we assume that the interest rate applying to such borrowings also equals r. Finally, consumers have access to credit cards. Outstanding balances on a credit card can be increased at any time by using the card to finance purchases. In addition, consumers can

[14] Hartley (1995) analyzes a model in which credit cards are the only form of unsecured borrowing available to consumers. The model also allows for a grace period on credit card debits. If consumers do not have card liabilities at the beginning of the period and also have other savings in excess of their card liabilities at the end of the period, then interest is not payable on their card debits during the period. It is shown that it is rational for many consumers to incur interest charges on credit card liabilities despite extremely high credit card interest rates relative to the interest rates applying to savings. The model discussed in that paper cannot be solved analytically, however, and discussion of it is beyond the scope of the present paper.

change credit card balances when they choose their net loans for the period. They are charged an interest rate $i > r$ on credit card balances outstanding at the end of period t. Consumers also earn income y per period, but income earned in period t cannot be used to finance consumption during period t. An insolvency constraint requires wealth to exceed $-y/r$ for all periods t.

We consider the consumer's maximization problem at the time of asset market transactions for period t. Assume that wealth at that time is W_t in real terms. Let m_t denote real money balances, l_t real loans, and b_t real credit card balances outstanding at the end of asset market transactions for period t. During period t, consumers use their money balances and, possibly, new credit card debits d_t to finance consumption.

Define the consumer value function $V(W_t)$ to be the maximized expected discounted value of utility at the time of asset market transactions for period t. The function $V(W_t)$ satisfies the Bellman equation

$$V(W_t) = \max_{m_t, l_t, b_t} \beta\left\{\frac{1}{\Theta_R}\int_{\Theta_1}^{\Theta_2} \max_{c_t, d_t} [U(c_t, \theta_t) + V(W_{t+1})]d\theta\right\}, \quad (32)$$

where the maximizations are carried out subject to the constraints

$$m_t + l_t - b_t = W_t, \quad (33)$$

$$m_t \geq 0, \quad (34)$$

$$b_t \geq 0, \quad (35)$$

$$c_t \leq m_t + d_t, \quad (36)$$

$$d_t \geq 0, \quad (37)$$

and

$$\begin{aligned} W_{t+1} &= m_t - (c_t - d_t) + (1 + r)l_t - (1 + i)(b_t + d_t) + y \\ &= W_t + rl_t - i(b_t + d_t) + y - c_t. \end{aligned} \quad (38)$$

Consider first the choices of c_t and d_t taking m_t, l_t, b_t, and θ_t as given. Let the multipliers on the constraints (36) and (37) be μ_1 and μ_2, respectively, and define the Lagrangian

$$\begin{aligned} L = {} & U(c_t, \theta_t) + V[m_t - c_t + d_t + (1 + r)l_t - (1 + i)(b_t + d_t) + y_t] \\ & + \mu_1(m_t + d_t - c_t) + \mu_2 d_t. \end{aligned}$$

The first-order conditions for a maximum of L with respect to c_t and d_t are

$$U_c = \mu_1 + V', \quad (39)$$

$$\mu_1 + \mu_2 = iV', \quad (40)$$

and

$$\mu_2 d_t = 0, \quad \mu_2 \geq 0, \quad d_t \geq 0. \tag{41}$$

Let the solution to this problem be $L(m_t, l_t, b_t, \theta_t)$, and observe from the envelope theorem that

$$L_1 = V' + \mu_1, \tag{42}$$

$$L_2 = (1 + r)V', \tag{43}$$

and

$$L_3 = -(1 + i)V'. \tag{44}$$

Using the function L and defining multipliers λ for the budget constraint (33) and μ_3 and μ_4 for the inequalities (34) and (35), we can write the functional equation (32) as

$$\begin{aligned} V(W_t) = \max_{m_t, l_t, b_t} \beta\left[\frac{1}{\Theta_R}\int_{\Theta_1}^{\Theta_2} L(m_t, l_t, b_t, \theta_t)\,d\theta\right] \\ + \lambda(W_t + b_t - m_t - l_t) + \mu_3 m_t + \mu_4 b_t. \end{aligned} \tag{45}$$

If we denote the expectation of a function $Z(\theta)$ with respect to θ by $E_\theta[Z(\theta)]$, the first-order conditions for the choice of m_t, l_t, and b_t can be written

$$\beta E_\theta L_1 = \lambda - \mu_3 \quad \text{with } \mu_3 m_t = 0, \mu_3 \geq 0, m_t \geq 0, \tag{46}$$

$$\beta E_\theta L_2 = \lambda, \tag{47}$$

and

$$\beta E_\theta L_3 = -\lambda - \mu_4 \quad \text{with } \mu_4 b_t = 0, \mu_4 \geq 0, b_t \geq 0. \tag{48}$$

Now we can substitute equations (43) and (44) into equations (47) and (48) to obtain

$$\mu_4 = \beta E_\theta(i - r)V' > 0 \quad \text{for } i > r. \tag{49}$$

We conclude that when the interest rate on credit card balances exceeds the interest rate on loans, $b_t = 0$ and credit card debits will last no longer than a single period. While trading in the asset markets, consumers would always borrow at the interest rate r to liquidate any outstanding credit card liabilities when $i > r$. More generally, credit constraints could limit access to low-interest loans for many consumers so that credit card liabilities could last for many periods. Alternatively, as we have argued above, consumers could choose to carry positive credit card balances for several periods when there are transactions costs associated with arranging low-interest bank loans.

When equations (42) and (43) are substituted into (46) and (47), we obtain

$$\beta E_\theta \mu_1 + \mu_3 = \beta r E_\theta V' > 0. \tag{50}$$

From equation (50) we can conclude either that $\mu_3 > 0$ and real money balances chosen at the beginning of period t are zero or else that $\beta E_\theta \mu_1 > 0$, so that consumers anticipate being short of money in some states in period t.

While consumers will not begin period t with positive credit card balances when $i > r$, many of them will nevertheless choose to borrow on their credit cards to finance some consumption during the period. Access to credit cards, or "borrowing on demand," will enable consumers to hold low money balances and yet still be able to consume a large amount in the event that θ turns out to be relatively large.

Now suppose that θ is low enough that consumers are not short of money during period t. Then $c_t < m_t + d_t$ and $\mu_1 = 0$, and equation (40) implies $\mu_2 = iV' > 0$. Then, from equation (41), we can conclude that $d_t = 0$. Thus consumers will resort to credit cards to finance consumption only after they have exhausted the money balances they arranged to have on hand for the period.

If consumers do hold positive money balances at the beginning of period t, then $\mu_3 = 0$ and (50) implies $E_\theta \mu_1 = rE_\theta V'$. If we integrate (40) with respect to the distribution of θ, we obtain $E_\theta \mu_1 + E_\theta \mu_2 = iE_\theta V'$. Thus *the ratio of the expected liquidity value of cash balances to the expected liquidity value of credit card debits should equal* r/i.

Thus far, we have not used the assumption that the single-period utility function displays constant relative risk aversion. In this special case it is relatively easy to guess the functional forms for the value function V and the maximizing choices of m_t, l_t, and d_t.

PROPOSITION. For the utility function specified in (31), $V(W_t)$ is given by

$$V(W_t) = \frac{A}{1-\gamma}\left(\frac{y}{r} + W_t\right)^{1-\gamma} \tag{51}$$

for a constant A. Furthermore, the maximizing choice of real money balances is

$$m_t = B\left(\frac{y}{r} + W_t\right) \tag{52}$$

for another constant B. Credit card debits are zero for $\theta \leq \theta^*$, and for $\theta \geq \theta^*$ they are given by

$$\frac{(1-B)(1+r)[\theta/(1+i)A]^{1/\gamma} - B}{1 + (1+i)[\theta/(1+i)A]^{1/\gamma}}\left(\frac{y}{r} + W_t\right), \tag{53}$$

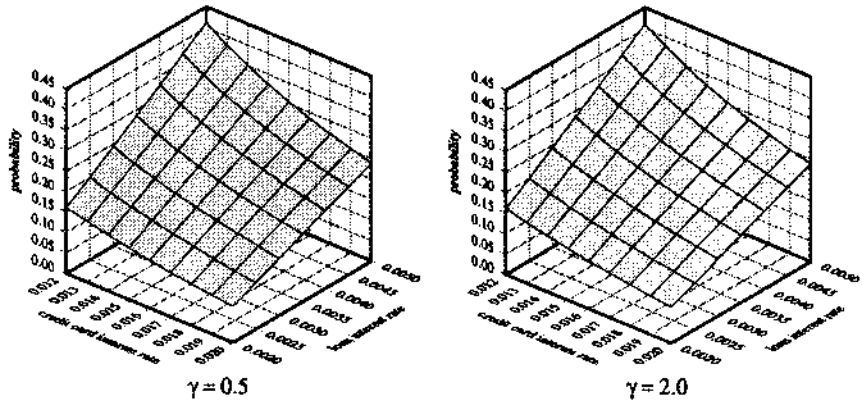

FIG. 2.—Probabilities of card use

where θ^* is defined by

$$\theta^* = (1 + i)A\left[\frac{B}{(1 + r)(1 - B)}\right]^{\gamma}. \tag{54}$$

Proof. The proof is given in the Appendix.

We solved the model numerically for a range of interest rates r and i and for two values for the coefficient of relative risk aversion, γ, namely $\gamma = 0.5$ and $\gamma = 2$. The range of values for the consumption shock θ was set at $[1, 5]$. As in table 3, we imagined a period as corresponding to a month of calendar time. Accordingly, we set the time discount factor β to 0.9965, which corresponds to a time discount rate of about 4.3 percent per year. We also examined the effects of a change in β and a mean-preserving spread in the distribution of θ.

Figure 2 graphs the solutions for probabilities $(\Theta_2 - \theta^*)/\Theta_R$ of using credit cards to finance purchases. These probabilities are all only slightly below the corresponding probability in the simple cost minimization model, r/i. For a high ratio of r to i, less risk averse consumers (with $\gamma = 0.5$) are slightly more likely to use their credit cards; for a low ratio of r to i, increased risk aversion marginally increases the probability of credit card use. A mean-preserving spread in the distribution of θ to $[0.25, 5.75]$ magnifies these differences by a very small amount.

Figure A1 in the Appendix graphs the solutions for the ratio B of cash balances to "total wealth" $([y/r] + W)$ and the ratio $\bar{D}$ of average credit card debits to total wealth for the same values of γ and interest rates. The per capita demands for real money balances and credit card debits, along with the average per capita consumption, depend on the cross-sectional wealth distribution in addition to the coefficients graphed in the figures. However, the ratios of the coefficients B and $\overline{D}$ indicate the *relative* magnitudes of the demands for the two types of assets.

We can compare the relative demands for card debits and cash with the results from the cost minimization model of the previous section. For $r = 0.0045$ and c distributed lognormally with standard deviation 1,500, credit card debits in table 3 range between 32 percent and 62 percent of real money balances as i declines from 0.018 to 0.012. By contrast, for $r = 0.0045$ and $\gamma = 0.5$, average card debits in the utility maximization problem range between 6 percent and 30 percent of real money balances as i declines from 0.018 to 0.010. One explanation of the difference is that, unlike the uniform distribution for θ assumed here, the lognormal distribution is skewed toward high values of c. However, an increase in the coefficient of relative risk aversion from $\gamma = 0.5$ to $\gamma = 2.0$ substantially reduces the ratio of credit card debits to real money balances.[15] Since the *probabilities* of using credit cards are not very sensitive to γ, this fall in average debits primarily arises because consumption responds less to increases of θ above θ^* when γ is higher. This can also be seen from (53), which implies that credit card debits become insensitive to θ as $\gamma \rightarrow \infty$.

In order to calculate the elasticities of average credit card debits with respect to i and r, we would also need to know the effect of changes in these interest rates on the equilibrium cross-sectional wealth distribution. However, we can calculate the elasticities of the average demand for credit card debits *relative to* money balances ($\overline{D}/B$). For the range of interest rates represented in the figures above, the elasticities with respect to r/i average 2.58 when $\gamma = 0.5$ and 2.41 when $\gamma = 2.0$.

VII. Market Equilibrium

Our analysis of the consumer maximization problem has shown that credit card debits of rational consumers depend on the ratio of the interest rate on competitive assets to the credit card interest rate. The demand for card debits is therefore likely to be less sensitive to a change in the credit card interest rate than to changes in interest rates on competitive assets. This result affects the relationship between the credit card interest rate and the cost of funds. A reduction in the cost

[15] When $\gamma = 2.0$, average debits as a percentage of real money balances range from 1.5 percent to 6 percent for $r = 0.0045$ and as i declines from 0.018 to 0.010. A spread in the distribution of θ to [0.25, 5.75] raises the percentages when $\gamma = 0.5$ to about 8 percent and 41 percent.

of funds is likely to be accompanied by falls in both the interest rate on loans that are competitive with credit cards and the opportunity cost of holding precautionary money balances. Since credit card debits respond more to these alternative interest rates than they do to the interest rate on credit cards, the demand for card debits from each existing consumer would fall following a uniform reduction in *all* interest rates. A drop in the cost of funds is therefore unlikely to be met by an increase in demand from existing customers, even if the interest rate on card debits were to fall to keep bank profits unchanged. Rather, the banks are likely to seek out additional customers and expand demand at the extensive margin. These additional customers, however, are likely to have a higher probability of default, and the interest rate on credit card debits will reflect this higher risk.

We can illustrate this argument with the following simple model. Throughout this discussion, we ignore credit card users who pay off their accounts every month. They are buying a different type of service. For simplicity, we also ignore the costs of processing credit card transactions and the income from merchants' fees. Insofar as the banks break even for those customers who do not borrow against their credit cards, the net effects of ignoring these components of costs and revenue will be negligible.

Until Section VII*B*, we assume that all banks charge the same credit card interest rate, i. For simplicity, we use $r < i$ to denote the cost of funds to the banks, the rate at which the bank discounts future revenue, the opportunity cost of money balances, and the interest rate on alternative financing for consumers. We assume that each bank takes i and r as given.

The assumption that i is effectively beyond the control of an individual bank needs some justification. Even though the credit card market is characterized by a large number of banks supplying products that are almost perfect substitutes, transactions costs limit the amount of switching between credit cards. Nevertheless, the banks have limited freedom to vary their rates. As we noted in Section IV, a bank that reduces its rate below the prevailing rate will tend to attract more big borrowers, or individuals who expect to be big borrowers, and these individuals are also likely to have a higher default risk. Conversely, a bank that raises its rate above the prevailing rate will tend to lose individuals who have reasonably large balances and yet remain well within their credit limit and therefore can easily transfer balances to other cards. These are some of the most desirable customers from the bank's point of view.[16] We assume that the bank

[16] Klemperer (1987) analyzes switching costs in a two-period model with two firms engaged in price competition in the first period but with consumers tied to suppliers

expects to lose a fraction h of its customers each period to other banks and for reasons other than default.

Our analysis of the consumer maximization problem implies that the demand for credit card financing can be written as a function $D(r/i)$ of the ratio of the interest rates on competitive assets to the credit card interest rate with $D' > 0$. Let π be the expected rate of growth of *nominal* balances outstanding. For notational convenience define the real discount factor ρ by $1/(1 + \rho) = (1 + \pi)/(1 + r)$. The expected discounted revenue from a customer with default probability p and current outstanding card balances D will then be

$$[D(i - r)(1 - p) - Dp]\left[1 + \sum_{t=1} \frac{(1 - h)^t(1 - p)^t}{(1 + \rho)^t}\right] = \frac{D[(i - r)(1 - p) - p](1 + \rho)}{\rho + h + p(1 - h)}. \tag{55}$$

Assume that banks can, at a cost c, observe a vector of characteristics of individuals—such as age, number of dependents, education, income, credit history, and employment history—that indicate the probability that the individual will default. Suppose that the characteristics can be incorporated as a weighted vector into the variable n, with $p = p(n)$ denoting the expected default probability of an individual with weighted characteristics equal to n.[17] Until Section VIIB, we assume that the group of applicants for a credit card at each bank is representative of the population as a whole. After processing an application and calculating p, the bank can either grant or deny the application for a credit card. Suppose that the qualifying level of expected default probability, p^*, is chosen to maximize expected discounted profits per customer:

$$\int_0^{p^*} \frac{D\{(i - r)[1 - p(n)] - p(n)\}(1 + \rho)}{\rho + h + p(n)(1 - h)} f(p)\,dp - c, \tag{56}$$

where $f(p)$ is the proportion of the population (excluding card holders who always pay off their balances in full) with expected default probability p. The first-order condition for a maximum with respect

in the second period. However, his model is not directly applicable to the credit card market, where there are a large number of firms and periods. Further, many credit card customers hold more than one card and face trivial switching costs. There is also adverse selection in price competition and a "lemons problem" since unprofitable customers may be returned to the pool.

[17] Greene (1992) observes that banks use linear functions of the vector of characteristics revealed in loan applications and credit bureau inquiries to estimate the default probability of potential customers and then grant or deny the application.

to p^* is

$$\frac{D[(i-r)(1-p^*)-p^*](1+\rho)}{\rho+h+p^*(1-h)}f(p^*)=0, \tag{57}$$

and this requires

$$i-r=\frac{p^*}{1-p^*}. \tag{58}$$

The economywide per capita demand for credit card funds from interest-paying customers will be

$$D\left(\frac{r}{i}\right)\int_0^{p^*}f(p)\,dp=D\left(\frac{r}{i}\right)F(p^*), \tag{59}$$

where $F(p)$ is the distribution function corresponding to $f(p)$. We assume that the per capita supply of funds to credit card borrowers, $S(r)$, is a decreasing function of the cost of funds r. Thus, in equilibrium, i and p^* will be determined by (58) and

$$D\left(\frac{r}{i}\right)F(p^*)=S(r). \tag{60}$$

We are interested primarily in the effects on equilibrium i of a change in r. Totally differentiating (58) and (60), we obtain

$$\begin{bmatrix} 1 & -\dfrac{1}{(1-p^*)^2} \\ \dfrac{-rD'F(p^*)}{i^2} & Df(p^*) \end{bmatrix}\begin{bmatrix} \dfrac{di}{dr} \\ \dfrac{dp^*}{dr} \end{bmatrix}=\begin{bmatrix} 1 \\ S'-\dfrac{D'F(p^*)}{i} \end{bmatrix}. \tag{61}$$

The determinant on the left of (61) can be evaluated as

$$\det=Df(p^*)-\frac{rD'F(p^*)}{i^2(1-p^*)^2}. \tag{62}$$

Since $D'>0$, the sign of det is indeterminate. However, from (58), det can also be written as

$$Df(p^*)\left[1-\frac{rD'}{iD}\frac{F(p^*)}{p^*f(p^*)}\frac{(1+i-r)(i-r)}{i}\right]. \tag{63}$$

From the analysis of the consumer maximization problem, we would not expect the elasticity of D with respect to r/i to exceed 1.0.[18] The

[18] The elasticities of average card debits with respect to r/i in table 3 are about 0.62 in the lognormal case and 1.0 in the negative exponential case. The larger elasticities

average credit card interest rates in the Federal Reserve series for the 1980s are about 18.3 percent, whereas the cost of funds, as measured by the 6-month bank certificate of deposit rate, averaged 9.0 percent in the same period. Setting $i = 0.183$ and $r = 0.09$, we find $(1 + i - r)(i - r)/i \approx 0.555$.

It is more difficult to assign likely magnitudes to the elasticity of F with respect to p. If we assume that $\ln[p/(1 - p)]$ is normally distributed, however, the elasticity can be estimated using information on the average probability of default and the proportion of households holding credit cards.

Ausubel (1991) reports the average default probability in his table 4 to be about 2.75 percent for the late 1980s. An article in the *New York Times* ("Banks Uneasy at Focus on Credit Cards" [November 19, 1991]) gives the losses from bankruptcy and fraud in 1991 as \$4.26 billion. The March 1992 *Nilson Report* (HSN Consultants, Los Angeles) gives the total outstanding balances of bank credit card issuers in 1991 as \$167.42 billion. Thus losses from bankruptcy and fraud amounted to about 2.54 percent of outstanding balances in 1991. However, the *New York Times* article also lists losses from late payment and nonpayment of card balances of \$4.11 billion in 1991. This is almost as much as the losses from bankruptcy and fraud and could be incorporated into our simple model by taking the average default probability to be slightly higher than 2.75 percent, say 3 percent. Since the 25 percent of customers who always pay off their accounts in full have a zero default probability, this would imply an average default probability for the remaining 75 percent of customers of 4 percent.[19]

If we again take $i = 0.183$ and $r = 0.09$, (58) implies $p^*/(1 - p^*) = .093$ and hence $p^* = .085$. Calem and Mester (1993, p. 11) report that in the 1989 Survey of Consumer Finances sponsored by the Federal Reserve, 1,665 households out of 3,143 (i.e., about 53 percent of households) had at least one bank-type credit card.[20] This would

calculated at the end of Sec. VI were the elasticities of the demand for card debits *minus* the demand for cash. The assumption of a uniform distribution for θ in Sec. VI probably also raises the elasticities of demand relative to a "more realistic" assumption of a negative exponential or lognormal distribution. The opportunity to respond to interest rate changes by varying consumption levels in addition to the method of financing should *reduce* elasticities of demand for card debits relative to those applying in the simple cost minimization model.

[19] Greene (1992, p. 12) reports that "default rates on credit cards are quite small, usually ranging from one to five percent depending on the stratum sampled." The average default probability in his sample is 10 percent, and in order to "match more closely [its] population counterpart," he scales his data so that the average default probability is 5 percent.

[20] Greene (1992, p. 17) reports that "in the population, the proportion of card applications which are accepted is not more than half."

imply that households that always pay off their accounts in full were about 13 percent of the population, and those paying interest were about 40 percent of the population. When the households that always pay off their accounts in full are excluded, this would imply that the probability that $p \leq p^*$ for the remaining households was about 45 percent.

If we assume that $\ln[p/(1-p)]$ is normally distributed with mean μ and variance σ^2 and transform variables from $p/(1-p)$ to p, the density function of p is given by

$$f(p) = \frac{1}{p(1-p)\sigma\sqrt{2\pi}} \exp\left\{-\frac{[\ln p - \ln(1-p) - \mu]^2}{2\sigma^2}\right\}. \tag{64}$$

We then get two equations to solve for μ and σ:

$$N\left[\frac{\ln(0.093) - \mu}{\sigma}\right] = 0.45 \tag{65}$$

and

$$\begin{aligned} E\left(p \,\middle|\, \frac{p}{1-p} \leq 0.093\right) &= \frac{1}{0.45\sigma\sqrt{2\pi}} \int_0^{0.085} \frac{1}{1-p} \\ &\times \exp\left\{-\frac{[\ln p - \ln(1-p) - \mu]^2}{2\sigma^2}\right\} dp = 0.04, \end{aligned} \tag{66}$$

where N is the cumulative distribution function for the standard normal random variable. These equations can be solved for $\mu = -2.2029$ and $\sigma = 1.37078$. The density function (64) with these values of μ and σ is graphed in figure 3. The elasticity of $F(p)$ with respect to p, evaluated at $p^* = .085$, then works out to approximately 1.426. The implied value for the term in brackets in (63) is therefore greater than $1 - (1.426 \times 0.555) = 0.208$, and we conclude that det is likely to be positive.

We conclude from (61) that

$$\frac{di}{dr} = 1 + \frac{[(r-i)D'F(p^*)/i^2] + S'}{\det} \frac{1}{(1-p^*)^2} \tag{67}$$

and

$$\frac{dp^*}{dr} = \frac{S' - D'F(p^*)}{\det}, \tag{68}$$

and for $D' > 0$, $S' < 0$, and $\det > 0$, $di/dr < 1$ and $dp^*/dr < 0$. Thus, when a fall in r increases p^*, the fall in r also results in a *smaller* fall in i than in r.

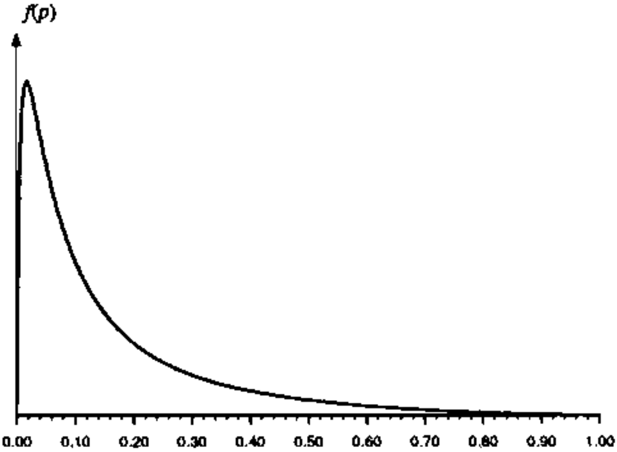

FIG. 3.—Distribution of default probability

The sign of di/dr is actually ambiguous, so that a fall in the cost of funds r could *raise* the equilibrium credit card interest rate. The part of di/dr that does not involve S' can be written

$$\left[1 - \frac{rD'}{iD}\frac{F(p^*)}{p^*f(p^*)}\frac{(1+i-r)(i-r)}{r}\right] \Big/ \left[1 - \frac{rD'}{iD}\frac{F(p^*)}{p^*f(p^*)}\frac{(1+i-r)(i-r)}{i}\right]. \quad (69)$$

If we again assume that the elasticity of D with respect to r/i is 1.0 and use $i = 0.183$, $r = 0.09$, and the distribution above for p, then (69) is negative. However, if the elasticity of D with respect to r/i falls below 0.62, expression (69) becomes positive. Even if $di/dr > 0$, we would expect to find it considerably below 1.0.

A simple competitive model of the credit card industry therefore suggests that a fall in the cost of funds is likely to produce an increase in the number of customers of each bank, an increase in the probability of default at all banks, and at most a slight reduction in credit card interest rates. A crucial feature of the model required to obtain these results is the assumption, based on the analysis of the consumer problem, that D is a function of r/i with $D' > 0$.[21]

[21] We examined the relationship between average credit card interest rates and the cost of funds using several series from the *Federal Reserve Bulletin* (covering the period 1982.II–1993.I). The 6-month bank CD rate lagged one quarter appeared to be slightly better than the lagged 1-year Treasury bill rate for explaining movements in the credit card interest rate. Both the credit card interest rate i_t and the 6-month bank CD rate r_t also appeared to be nonstationary with i_t and r_{t-1} cointegrated. The estimated long-run, or cointegrating, relationship was $i_t = 17.34 + 0.105r_{t-1}$. This implies a long-run *spread* between i_t and r_{t-1} of 17.34 percentage points *minus* $0.895r_{t-1}$. That is, the cointegrating relationship implies that the spread *increases* as the cost of funds

A. Interbank Sales of Credit Card Accounts

We can use the idea that i will reflect the default probability of the *marginal* credit card holder to explain the evidence Ausubel (1991, p. 66, table 9) presents showing that, in the late 1980s, credit card accounts sold for premia between 3 percent and 27 percent with an average of around 22 percent. Ausubel finds that "it is difficult to justify the recent flurry of premia in the range of 23–27 percent unless returns equaling at least three times the ordinary rate of return in banking are expected to persist" (p. 67). He can justify the premia only by assuming in addition that outstanding balances on the credit card accounts have very high expected growth rates. Our theory can explain the observed premia assuming that growth in the outstanding balances on the accounts is expected to match only the rate of inflation.

By purchasing credit card accounts from another bank, a purchasing bank can avoid the cost c of processing an application. Thus if we substitute (58) into the expression (56) for bank profits and eliminate the cost c/D, we can write the discounted expected profits per unit of credit card debits for a representative group of credit card holders (who all have default probability less than or equal to p^*) as

$$\frac{1 + \rho}{1 - p^*} \int_0^{p^*} \frac{p^*(1 - p) - p(1 - p^*)}{\rho + h + p(1 - h)} \frac{f(p)}{F(p^*)} dp. \tag{70}$$

Then Ausubel's data imply that the discounted expected revenue per unit of credit card debits (70) should approximate 22 percent.

Recall that we have assumed that balances held by the average customer will remain constant in real terms, but that over time there will be an attrition of customers to other banks and as a result of defaults. If the half-life of the pool is T years, then we shall have

$$\int_0^{p^*} (1 - h)^T (1 - p)^T \frac{f(p)}{F(p^*)} dp = 0.5. \tag{71}$$

falls and the spread *decreases* as the cost of funds rises. The short-run dynamic equation (error correction mechanism) we estimated was (estimated standard errors in parentheses)

$$\underset{}{i_t - i_{t-1}} = \underset{(0.0137)}{-0.0106} + \underset{(0.1161)}{0.5245}(i_{t-1} - i_{t-2}) - \underset{(0.0451)}{0.1332}(i_{t-1} - 17.34 - 0.105 r_{t-2}) + u_t,$$

with $R^2 = .47$, and the Q-statistic for eight lags is 3.41. When $r_{t-1} - r_{t-2}$ was included as a regressor, it had an estimated coefficient of 0.0249 with a standard error of 0.0140, so there was little evidence that r_{t-1} affected i_t apart from causing deviations from the long-run cointegrating relationship. The estimated dynamic equation implies that, following a permanent shock to r_t, i_t will be adjusted more than 30 percent toward its long-run value after 1 year and almost completely adjusted in 2 years.

Since ρ is the time discount rate of the bank after one allows for expected growth in the *nominal* balances outstanding, if $r = 0.09$ and the expected growth rate in D at that time was about 5 percent, we would have $\rho = .04/1.05$. If we use this value of ρ and the distribution for p calculated above, a premium of 22 percent on the accounts can be explained if $h = 0.172$, which corresponds to a half-life T of about 3.03 years.[22]

B. *Multiple Interest Rates and a Zero-Profit Equilibrium*

While the model above can explain some characteristics of the market for credit card loans, it is inconsistent with other features of the market. Contrary to our assumption that i is unique, several credit card interest rates coexist in the market, although there does not appear to be a continuum of rates on offer. In addition, since barriers to entry into the industry are apparently very low, expected discounted profits net of entry costs should be driven to zero for the marginal entrant.[23] We shall argue that the proliferation of interest rates limits the potential profits of new entrants into the industry.

Apart from the more detailed information on potential customers that banks obtain when they process a credit card application, banks have access to less informative, but more cheaply observed, signals about an individual's likely probability of default and demand for credit. For example, before it invites someone to apply for a credit card, a bank is likely to know that individual's zip code and other information extracted from computer databases such as club memberships, membership in professional organizations, magazine subscriptions, or ownership of assets such as a boat or a particular type of automobile. Of course, some cheaply observed characteristics such as gender, race, or national origin might be legally proscribed as a basis for discrimination.

Suppose that banks could use such *preliminary* information to classify potential interest-paying card holders into a number of risk categories. They would then invite individuals within a given category to *apply* for a card at a specified interest rate but *provide* cards to custom-

[22] Ausubel (1991) considers a range of values between 2 and 6 years for the *expected* life of the receivables. For a Poisson distribution, a half-life of 1 year corresponds to an expected life of 2 years, a half-life of 3 years to an expected life of 4.8 years, and a half-life of 4 years to an expected life of around 6.3 years.

[23] The market for bank credit cards in the United States has experienced considerable entry by new firms in recent years. For example, Ausubel (1991, p. 50) gives the number of firms in the industry as about 4,000 in 1988, whereas the 1992 *Nilson Report* gives the number of firms as about 6,000 in 1991.

ers solely on the basis of the more detailed information on income, credit history, and so forth revealed in the applications. The allocation of potential customers to risk categories will be determined endogenously by a set of zero-profit conditions. Banks will solicit applications from the various risk classes as long as the expected return from an additional application, net of processing costs, is positive. An equilibrium is reached when the pool of potential customers is depleted so that the expected return is zero. One factor reducing expected profits as more customers are removed from each pool is that individuals who have previously defaulted will become a more significant component of the pool. They will be returned to the pool when they default, and they will find it difficult to leave the pool since a previous default is an important component of the information banks examine when processing an application.[24] We can describe the equilibrium in algebraic terms as follows.

Let N be the fraction of the total population that is sampled by the credit card industry, K be the number of risk categories, and ϕ_k, $k = 1, 2, \ldots, K$, the proportion of the sampled population that is in category k. Note that since the ϕ_k are a probability distribution of customers across categories, we have

$$\sum_{k=1}^{K} \phi_k = 1. \tag{72}$$

Potential customers within group k would be invited to apply for a card at the interest rate i_k. As a result of the card switching costs discussed in Section IV, a new card is unlikely to attract an individual unless the interest rate is at least several percentage points below the rate on his existing card.[25] If we let i_k be the interest rate offered to potential customers in category k and i_{k-1} the next highest interest

[24] Greene (1992) reports several indicators of past delinquencies that banks obtain from credit bureaus and use as indicators of default probability when processing credit card applications. He estimates a probit equation for approval of an application and reports that "the most significant explanatory variables are the number of major derogatory reports and credit bureau inquiries (negative) and the number of open trade accounts (positive)" (p. 19).

[25] Our emphasis on credit card switching costs and the costs of obtaining information about the likely default probability of customers contrasts with the type of information and transaction costs that have been emphasized in the banking literature. Much of this literature has assumed an asymmetry of information between the lenders and the borrowers, who are typically thought of as investors in physical assets who have some control over the return on their investment. This leads to "costly state verification" and moral hazard problems as discussed by Holmstrom (1979) and Townsend (1979). This general idea has been applied to the banking industry by, e.g., Williamson (1986, 1987) and Bernanke and Gertler (1989). Diamond (1984) and Williamson (1986) have also emphasized the role of fixed costs and economies of scale in leading to a role for intermediaries as monitors of borrowers on behalf of lenders.

rate, no bank would find it worthwhile to offer a rate between i_k and i_{k-1} as long as

$$i_k - i_{k-1} \leq \tau(i_k), \tag{73}$$

where $\tau(i_k)$ represents the minimum reduction in rates below i_k that would be sufficient to offset the transactions costs of switching cards.

The banks will choose a critical value p_k^* of the expected default probability for each group of customers such that

$$i_k - r = \frac{p_k^*}{1 - p_k^*}, \quad k = 1, 2, 3, \ldots, K. \tag{74}$$

Only customers who have an expected default probability less than p_k^* *after* their application has been processed will be offered a card.

The distribution of expected default probabilities within each risk category k will depend on the fraction N of the population sampled by the industry and the allocation $\Phi = (\phi_1, \phi_2, \ldots, \phi_K)$ of potential consumers to risk categories. Write the distribution function for the kth category as $F_k(p, N, \Phi)$ and let the corresponding density function be $f_k(p, N, \Phi)$. We assume that banks can costlessly allocate potential consumers to the K risk categories. With freedom of entry into any one of the K credit card markets, the potential customers will have to be allocated in such a way that the expected profits obtainable from any new consumer chosen from group k, and therefore offered a card at rate i_k, are nonnegative, and the expected profits would be negative were that consumer to be offered a card at an interest rate less than i_k. The following K zero-profit conditions will therefore have to hold:

$$\frac{D(r/i_k)(1 + \rho)}{1 - p_k^*} \int_0^{p_k^*} \frac{p_k^*(1 - p) - p(1 - p_k^*)}{\rho + h + p(1 - h)} f_k(p, N, \Phi)\, dp = c, \quad k = 1, 2, 3, \ldots, K. \tag{75}$$

Finally, if we again denote the per capita supply of funds to credit card borrowers by $S(r)$, the market equilibrium condition (60) would be replaced by

$$N \sum_{k=1}^{K} \phi_k D\left(\frac{r}{i_k}\right) F_k(p_k^*, N, \Phi) = S(r). \tag{76}$$

Equation (72), equations (73) holding as equalities, and equations (74), (75), and (76) would then provide $1 + (K - 1) + K + K + 1 = 3K + 1$ equations to determine the $3K + 1$ endogenous variables $i_1, i_2, \ldots, i_K, p_1^*, p_2^*, \ldots, p_K^*, \phi_1, \phi_2, \ldots, \phi_K$, and N.

This configuration of interest rates, and allocation of potential customers, would constitute an equilibrium in the industry since no existing supplier would have an incentive to deviate and offer a different set of rates or choose a different level for any of the p_k^*, and a new entrant to the industry could not make positive expected profits. Given the transaction costs of switching cards, the only customers a bank could attract with an interest rate below i_k but above i_{k-1} would be those in category $k + 1$ or higher. Since the bank knows nothing about these customers except the category to which they belong, the expected profits obtainable from them at an interest rate below i_k would be negative.[26]

Since the equilibrium number of credit card interest rates depends on the spread between i_1 and r, a fall in the cost of funds to the credit card industry would be associated not only with an expansion in the total number of credit card holders but also with an increase in the number of interest rates on offer. Average credit card interest rates therefore would fall. Nevertheless, credit cards would be extended to riskier customers *within* each risk category following a drop in the cost of funds. The spread between i_k and the cost of funds r within each category would still rise as r falls, so that average credit card interest rates would still appear relatively inflexible.

Although the zero-profit condition is satisfied in equilibrium, existing credit card portfolios would still sell at a premium that reflects the cost of assembling the portfolios in addition to any rents that may accrue to early entrants in the market.[27] The ability to use low-

[26] Even if the system of equations has a unique solution, the equilibrium configuration of credit card interest rates, and allocation of consumers, will not be unique. Any set of interest rates i_k that satisfies (72) as a set of *inequalities*, along with its corresponding set of critical values p_k^* and allocation of potential consumers given by N and Φ, will also be an equilibrium. Thus the configuration of interest rates a market starts with will influence the final configuration achieved after a change in the cost of funds.

[27] The premium reflecting the costs of assembling a portfolio of accounts should, however, amount to no more than about 5 percent. The *New York Times* article mentioned above shows that the total costs (excluding taxes) for the U.S. bank card industry were \$28.41 billion in 1991. Apart from the cost of funds (\$12.55 billion) and losses from late/nonpayment, bankruptcies, and fraud (\$8.37 billion), the remaining costs were overhead and data processing (\$2.32 billion), advertising (\$0.49 billion), collection (\$2.10 billion), and other operating expenses (\$2.58 billion). If we subtract annual fees (\$2.38 billion), fees from merchants (\$3.56 billion), and miscellaneous income (\$0.99 billion) from these costs, we are left with an estimate of \$560 million for cN, where N is the number of credit card applications processed. The February 1992 *Nilson Report* gives the total number of new bank credit card accounts for the top 50 account holders in 1991, who have 75 percent of all accounts, as 11.98 million. This suggests a value for N of at least 16 million, implying that c is at most \$35. The March 1992 *Nilson Report* shows that the average outstanding balance on active accounts with the top 100 issuers in 1991 was about \$1,500. These numbers imply that c/D is approximately 2.3 percent. The average value for the proportion of applications approved is about 50 percent. Hence, if the portfolio is a representative selection of accounts of individuals *guaranteed* to qualify, and therefore guaranteed to have $p \leq p_k^*$, the average return per unit of outstanding debits should approximate $2c/D$.

cost information to separate customers into different risk categories should, however, limit the range of default probabilities within any single risk category and therefore also limit the rents accruing to early entrants in the market.

VIII. Conclusion

Credit cards are very useful as a low-cost method of financing transactions and arranging short-term loans. Despite large interest rates, rational consumers would frequently pay interest on outstanding credit card balances rather than pay the transaction costs associated with arranging loans from banks or other financial institutions. A rational consumer may also pay interest on credit card debits to avoid some of the costs associated with holding precautionary money balances.

We have also argued that competitively determined credit card interest rates are not likely to be very responsive to changes in the cost of funds. A change in the cost of funds is correlated with a change in other interest rates, and in particular the opportunity cost of money and the interest rate on loans competitive with credit cards. The demand for credit card debits from a rational consumer is more sensitive to these alternative interest rates than to the credit card interest rate. In consequence, an increase in the supply of funds to the credit card business that is accompanied by a reduction in interest rates will decrease the amount debited to each card but increase the number of individuals using credit cards. Since the costs of serving the additional customers are expected to be higher, there will be little pressure for credit card interest rates to fall.

Finally, we showed that relatively inflexible credit card interest rates can be consistent with a competitive equilibrium in which entry costs are very low, and a marginal entrant to the industry earns zero profits. Premiums of about 5 percent for portfolios of credit card receivables are also consistent with an equilibrium in which *all* banks earn zero profits. The relatively large premiums that appear to have been paid for some portfolios of credit card receivables in the recent past, however, probably reflect the fact that not all banks have identical pools of customers. As long as it is costly for customers to signal that they are large but low-risk borrowers, early entrants who have selected a group of customers with a relatively low default probability may be able to retain, and earn rents on, many of those customers. These rents are capitalized into the sale value of the portfolio of receivables.

Appendix

Proof of the Proposition in Section VI

Substitute the functional form for $U(c_t, \theta_t)$, the guessed functional form for $V(W_t)$, and the budget constraint (38) with $b_t = 0$ into the first-order conditions (39) and (40) to obtain

$$\theta c_t^{-\gamma} - A\left[\frac{y}{r} + m_t + (1 + r)l_t - id_t + y - c_t\right]^{-\gamma} - \mu_1 = 0 \tag{A1}$$

and

$$-iA\left[\frac{y}{r} + m_t + (1 + r)l_t - id_t + y - c_t\right]^{-\gamma} + \mu_1 + \mu_2 = 0. \tag{A2}$$

Now for $c_t < m_t + d_t$, $\mu_1 = 0$, $\mu_2 > 0$, $d_t = 0$ and, from (A1), c_t is given by

$$c_t = \frac{(\theta/A)^{1/\gamma}[(y/r) + m_t + (1 + r)l_t + y]}{1 + (\theta/A)^{1/\gamma}}. \tag{A3}$$

On the other hand, if $c_t = m_t + d_t$ and $d_t > 0$, then $\mu_2 = 0$ and, from (A1) and (A2), $m_t + d_t = c_t$ is given by

$$\theta(m_t + d_t)^{-\gamma} = (1 + i)A\left[\frac{y}{r} + m_t + (1 + r)l_t - id_t + y - (m_t + d_t)\right]^{-\gamma}. \tag{A4}$$

Equation (A4) can be solved for d_t in terms of m_t and θ as follows:

$$\frac{[\theta/(1 + i)A]^{1/\gamma}[(y/r) + (1 + r)l_t + y] - m_t}{1 + (1 + i)[\theta/(1 + i)A]^{1/\gamma}}. \tag{A5}$$

Equation (A5) implies that $d_t > 0$ for $\theta > \theta^*$, where θ^* is given by[28]

$$\theta^* = (1 + i)A\left[\frac{m_t}{(y/r) + (1 + r)l_t + y}\right]^{\gamma}. \tag{A6}$$

Now substitute the functional forms for $U(c_t, \theta_t)$, the guessed functional form for $V(W_t)$, the budget constraint (33) with $b_t = 0$, and equations (A3) and (A5) for the maximizing choices of c_t and d_t into the functional equation (32) to obtain

$$\begin{aligned} V(W_t) = \max_{m_t, l_t} \frac{\beta}{(1 - \gamma)\Theta_R}\Bigg(&\int_{\Theta_L}^{\theta^*} A\left[1 + \left(\frac{\theta}{A}\right)^{1/\gamma}\right]^{\gamma}\left[\frac{y}{r} + m_t + (1 + r)l_t + y\right]^{1-\gamma} d\theta \\ &+ \int_{\theta^*}^{\Theta_2} A\left\{1 + (1 + i)\left[\frac{\theta}{(1 + i)A}\right]^{1/\gamma}\right\}^{\gamma} \\ &\times \left[\frac{y}{r} + (1 + i)m_t + (1 + r)l_t + y\right]^{1-\gamma} d\theta\Bigg). \end{aligned} \tag{A7}$$

[28] Observe from eqq. (A3) and (A5) that $c_t = m_t$ and $d_t = 0$ for a range of θ values, the size of which increases with the interest rate on credit card balances. As θ increases within this range, μ_2 decreases from a strictly positive number to zero.

The maximization in equation (A7) is carried out subject to the constraint $m_t + l_t = W_t$ and the functional relationship in equation (A6) between θ^*, m_t, and l_t. We can use the budget constraint to eliminate l_t from the maximization. Define the integrals

$$I_1 = \int_{\Theta_1}^{\theta^*} \left[1 + \left(\frac{\theta}{A}\right)^{1/\gamma}\right]^{\gamma} d\theta \tag{A8}$$

and

$$I_2 = \int_{\theta^*}^{\Theta_2} \left\{1 + (1+i)\left[\frac{\theta}{(1+i)A}\right]^{1/\gamma}\right\}^{\gamma} d\theta, \tag{A9}$$

and note that from (A6) and $m_t + l_t = W_t$,

$$\frac{d\theta^*}{dm} = \frac{\gamma\theta^*}{m_t}\,\frac{(y/r) + W_t}{(y/r) + W_t - m_t}. \tag{A10}$$

Then the first-order condition for the choice of m_t can be written

$$\begin{aligned}
&-r(1-\gamma)\left[\left(\frac{y}{r} + W_t\right)(1+r) - rm_t\right]^{-\gamma} I_1 \\
&\quad + (i-r)(1-\gamma)\left[\left(\frac{y}{r} + W_t\right)(1+r) + (i-r)m_t\right]^{-\gamma} I_2 \\
&\quad + \left[\left(\frac{y}{r} + W_t\right)(1+r) - rm_t\right]^{1-\gamma}\left[1 + \left(\frac{\theta^*}{A}\right)^{1/\gamma}\right]^{\gamma}\frac{d\theta^*}{dm} \\
&-\left[\left(\frac{y}{r} + W_t\right)(1+r) + (i-r)m_t\right]^{1-\gamma}\left\{1 + (1+i)\left[\frac{\theta^*}{(1+i)A}\right]^{1/\gamma}\right\}^{\gamma}\frac{d\theta^*}{dm} = 0.
\end{aligned} \tag{A11}$$

It is easy to verify that, with (A10), equation (A11) can be solved for m_t given by equation (52) for a constant B, which, along with A, solves

$$\begin{aligned}
&-r(1-\gamma)(1+r-rB)^{-\gamma}I_1 + (i-r)(1-\gamma)[1+r+(i-r)B]^{-\gamma}I_2 \\
&\quad + \left[1 + \left(\frac{\theta^*}{A}\right)^{1/\gamma}\right]^{\gamma}\frac{\gamma\theta^*(1+r-rB)^{1-\gamma}}{B(1-B)} \\
&\quad - \left\{1 + (1+i)\left[\frac{\theta^*}{(1+i)A}\right]^{1/\gamma}\right\}^{\gamma}\frac{\gamma\theta^*[1+r+(i-r)B]^{1-\gamma}}{B(1-B)} = 0,
\end{aligned} \tag{A12}$$

with θ^* given by equation (54). When we substitute the maximizing solution for m_t back into the functional equation (A7), we find that $V(W_t)$ is as specified in equation (51) if A and B are also chosen to satisfy the equation

$$\frac{\Theta_R}{\beta} = (1+r-rB)^{1-\gamma}I_1 + [1+r+(i-r)B]^{1-\gamma}I_2, \tag{A13}$$

where, from equations (A8), (A9), and (54), I_1 and I_2 are functions of A and B.

The integrals I_1 and I_2 are not easy to calculate for an arbitrary value of

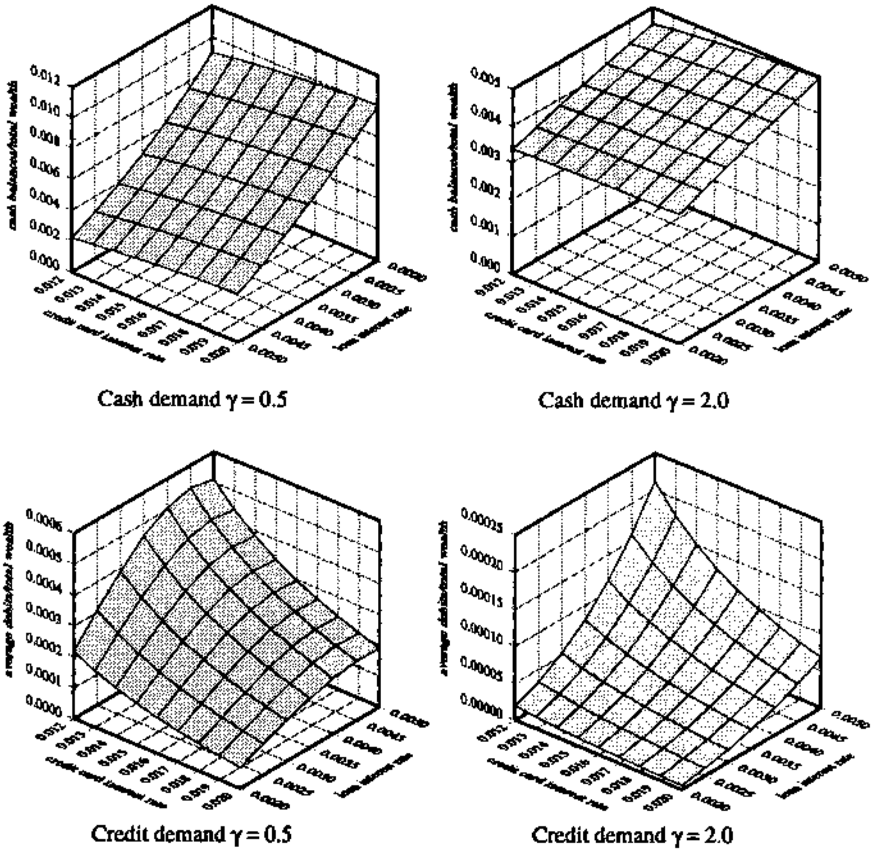

FIG. A1.—Demands for cash and card debits relative to wealth

the risk aversion parameter γ. For illustrative purposes, we examined results for $\gamma = 0.5$ and $\gamma = 2$. Figure A1 graphs the solutions for the ratio B of cash balances to "total wealth," $(y/r) + W$, and the ratio $\overline{D}$ of average credit card debits to total wealth.

When $\gamma = 0.5$, increases in the loan interest rate r reduce the ratio of cash balances to total wealth as average consumption falls. When γ is increased to 2.0, increases in interest rates tend to raise the ratio of average consumption to total wealth. These differing effects of a change in r on current consumption, with wealth held fixed, are what one might expect on the basis of the results from a simple two-period consumption maximization problem. Consumption and the demand for cash respond positively to changes in r when γ is larger since the substitution effects are much smaller when the utility function is more concave.[29]

[29] If we examine the problems of maximizing $\sqrt{c_t} + \beta\sqrt{c_{t+1}}$ or $-(1/c_t) - (\beta/c_{t+1})$ subject to $c_{t+1} = (W - c_t)(1 + r)$, the income and substitution effects of a change in r are such that, in the first case, $dc/dr < 0$ and, in the second case, $dc/dr > 0$.

An increase in the credit card interest rate i raises cash balances as consumers substitute toward cash as their preferred method of financing. When $\gamma = 0.5$, however, an increase in the loan rate r, with i held constant, initially raises card debits relative to total wealth, but eventually the decline in average consumption causes card debits to fall along with cash balances.

References

Ausubel, Lawrence M. "The Failure of Competition in the Credit Card Market." *A.E.R.* 81 (March 1991): 50–81.

Baumol, William J. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Q.J.E.* 66 (November 1952): 545–56.

Bernanke, Ben S., and Gertler, Mark. "Agency Costs, Net Worth, and Business Fluctuations." *A.E.R.* 79 (March 1989): 14–31.

Black, Fischer. "Banking and Interest Rates in a World without Money." *J. Bank Res.* 1 (Autumn 1970): 8–20.

Calem, Paul S., and Mester, Loretta J. "Search, Switching Costs, and the Stickiness of Credit Card Interest Rates." Manuscript. Philadelphia: Fed. Reserve Bank, 1993.

Diamond, Douglas W. "Financial Intermediation and Delegated Monitoring." *Rev. Econ. Studies* 51 (July 1984): 393–414.

Fama, Eugene F. "What's Different about Banks?" *J. Monetary Econ.* 15 (January 1985): 29–39.

Greene, William H. "A Statistical Model for Credit Scoring." Manuscript. New York: New York Univ., Dept. Econ., 1992.

Hartley, Peter R. "The Liquidity Services of Money." *Internat. Econ. Rev.* 29 (February 1988): 1–24.

———. "Value Function Approximation in the Presence of Uncertainty and Inequality Constraints: An Application to the Demand for Credit Cards." *J. Econ. Dynamics and Control* (1995), in press.

Hester, Donald D. "Monetary Policy in the 'Checkless' Economy." *J. Finance* 27 (May 1972): 279–93.

Holmstrom, Bengt. "Moral Hazard and Observability." *Bell J. Econ.* 10 (Spring 1979): 74–91.

Klemperer, Paul. "Markets with Consumer Switching Costs." *Q.J.E.* 102 (May 1987): 375–94.

Lucas, Robert E., Jr. "Money in a Theory of Finance." *Carnegie-Rochester Conf. Ser. Public Policy* 21 (Autumn 1984): 9–45.

Lucas, Robert E., Jr., and Stokey, Nancy L. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *J. Monetary Econ.* 12 (July 1983): 55–93.

———. "Money and Interest in a Cash-in-Advance Economy." *Econometrica* 55 (May 1987): 491–513.

Tobin, James. "The Interest-Elasticity of Transactions Demand for Cash." *Rev. Econ. and Statis.* 38 (August 1956): 241–47.

Townsend, Robert M. "Optimal Contracts and Competitive Markets with Costly State Verification." *J. Econ. Theory* 21 (October 1979): 265–93.

Williamson, Stephen D. "Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing." *J. Monetary Econ.* 18 (September 1986): 159–79.

———. "Financial Intermediation, Business Failures, and Real Business Cycles." *J.P.E.* 95 (December 1987): 1196–1216.